

A 4.5

$$(a) \quad Y(z) + 3 \cdot z^{-1} \cdot \{Y(z) + 1 \cdot z^{-1}\} = X(z)$$

$$Y(z) \cdot \{1 + 3z^{-1}\} = X(z) - 3$$

$$Y(z) = (1 + 3z^{-1})^{-1} \cdot \{X(z) - 3\}$$

$$X(z) = \frac{z}{z - \frac{1}{2}}$$

$$\rightarrow Y(z) = \frac{z}{z+3} \cdot \left( \frac{z}{z - \frac{1}{2}} - 3 \right) = z \cdot \left( \frac{z}{(z+3)(z - \frac{1}{2})} \right) - 3 \frac{z}{z+3} = A \frac{z}{z+3} + B \frac{z}{z - \frac{1}{2}} - 3 \frac{z}{z+3}$$

$$A = \frac{6}{7} \quad B = \frac{1}{7}$$

$$y[n] = \frac{6}{7}(-3)^n \delta[n] + \frac{1}{7} \left(\frac{1}{2}\right)^n \delta[n] - 3 \cdot (-3)^n \delta[n] = \left\{ \frac{1}{7} \left(\frac{1}{2}\right)^n - \frac{15}{7} \cdot (-3)^n \right\} \delta[n]$$

$$(b) \quad y[n] = \underbrace{y_{0E}[n]}_{=0} + y_{0Z}[n]$$

$$y[n] = y_{0Z}[n]$$

$$x[n] = \delta[n] \Rightarrow y[n] = \delta[n]$$

$$(c) \quad y_{0E}[n] \xrightarrow{\circ} Y_{0E}(z)$$

$$Y_{0E}(z) - \frac{1}{2} \cdot z^{-1} \{Y_{0E}(z) + 1 \cdot z^{-1}\} = 0$$

$$Y_{0E}(z) = \left(1 - \frac{1}{2} z^{-1}\right)^{-1} \cdot \frac{1}{2} = \frac{1}{2} \frac{z}{z - \frac{1}{2}} \xrightarrow{\circ} \frac{1}{2} \cdot \left(\frac{1}{2}\right)^n \delta[n]$$

$$y[n] = \frac{1}{2} \left(\frac{1}{2}\right)^n \delta[n] + \delta[n] = \left\{1 + \left(\frac{1}{2}\right)^{n+1}\right\} \delta[n]$$