

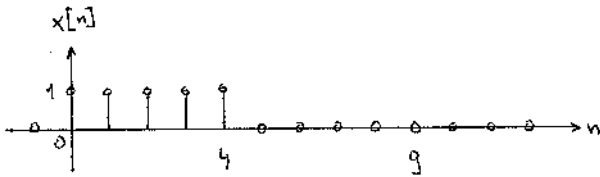
A 6.2

(a)  $N=10$ ,  $x[n] = \delta[n-5] \rightarrow X[k] = e^{-j\frac{2\pi}{10} \cdot 5 \cdot k} = (-1)^k$

(b)  $N=10$ ,  $x[n] = \delta[n+5] \rightarrow X[k] = (-1)^k$

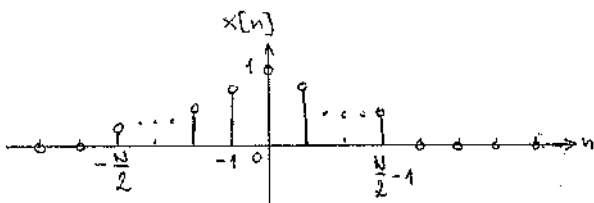
(c)  $N=20$ ,  $x[n] = e^{j\frac{2\pi}{20} \cdot 2 \cdot n} \rightarrow X[k] = 20 \cdot \delta[k-2]$

(d)  $N=10$ ,  $x[n] = x_1[n] - x_1[n-5]$



$$x[n] = x_1[n] - x_1[n-5] \rightarrow e^{-j\frac{2\pi}{10} \cdot 2} \frac{\sin(\frac{5\pi}{10}k)}{\sin(\frac{\pi}{10}k)} = e^{-j\frac{2\pi}{5}k} \frac{\sin(\frac{\pi}{2}k)}{\sin(\frac{\pi}{10}k)}$$

(e)



$N \in \mathbb{N}_0$

$$X[k] = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} x[n] e^{-j\frac{2\pi}{N}kn} = \sum_{n=-\frac{N}{2}}^{-1} \alpha^{-n} e^{-j\frac{2\pi}{N}kn} + \sum_{n=0}^{\frac{N}{2}-1} \alpha^n e^{-j\frac{2\pi}{N}kn} =$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \alpha^{-n-\frac{N}{2}} e^{-j\frac{2\pi}{N}kn} e^{j\frac{2\pi}{N}k\frac{N}{2}} + \sum_{n=0}^{\frac{N}{2}-1} \alpha^n e^{-j\frac{2\pi}{N}kn} =$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \alpha^{\frac{N}{2}} e^{j\pi k} \left( \alpha^{-1} e^{-j\frac{2\pi}{N}k} \right)^n + \sum_{n=0}^{\frac{N}{2}-1} \left( \alpha e^{-j\frac{2\pi}{N}k} \right)^n =$$

$$= \alpha^{\frac{N}{2}} (-1)^k \frac{1 - \alpha^{-\frac{N}{2} - j\pi k}}{1 - \alpha^{-1} e^{-j\frac{2\pi}{N}k}} + \frac{1 - \alpha^{\frac{N}{2} - j\pi k}}{1 - \alpha e^{-j\frac{2\pi}{N}k}} = \frac{1 - \alpha^{\frac{N}{2}} (-1)^k}{1 - \alpha e^{-j\frac{2\pi}{N}k}} + \alpha e^{j\frac{2\pi}{N}k} \frac{1 - \alpha^{\frac{N}{2}} (-1)^k}{1 - \alpha e^{j\frac{2\pi}{N}k}}$$