

$$(a) \quad Y(z) + \frac{1}{2} z^{-1} Y(z) + W(z) + z^{-1} W(z) = \frac{1}{2} X(z)$$

$$\underline{Y(z) - \frac{1}{2} z^{-1} Y(z) + 2W(z) - z^{-1} W(z) = -\frac{1}{2} X(z)}$$

$$2Y(z) + 3W(z) = 0$$

$$W(z) = -\frac{2}{3} Y(z)$$

$$Y(z) + \frac{1}{2} z^{-1} Y(z) - \frac{2}{3} Y(z) - \frac{2}{3} z^{-1} Y(z) = \frac{1}{2} X(z)$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{1}{2} \cdot \left(\frac{1}{3} - \frac{1}{6} z^{-1} \right)^{-1} = \frac{1}{2} \left(\frac{2z-1}{6z} \right)^{-1} = 3 \frac{z}{2z-1} = \frac{3}{2} \frac{z}{z - \frac{1}{2}}$$

$$H(e^{j\theta}) = \frac{3}{2} \frac{e^{j\theta}}{e^{j\theta} - \frac{1}{2}} = \frac{3}{2} \frac{1}{1 - \frac{1}{2} e^{-j\theta}}$$

$$h[n] = \frac{3}{2} \cdot \left(\frac{1}{2} \right)^n \delta[n]$$

$$(b) \quad Y(z) = H(z) \cdot X(z) = \frac{3}{2} \frac{z}{z - \frac{1}{2}} X(z) = \frac{3}{2} \cdot \frac{1}{1 - \frac{1}{2} z^{-1}} X(z)$$

$$Y(z) - \frac{1}{2} z^{-1} Y(z) = \frac{3}{2} X(z)$$

↓

$$y[n] - \frac{1}{2} y[n-1] = \frac{3}{2} x[n]$$

$$(c) \quad y[n] = \frac{1}{2} y[n-1] + \frac{3}{2} x[n] = \frac{1}{2} \cdot \{ y[n-1] + 3x[n] \}$$

