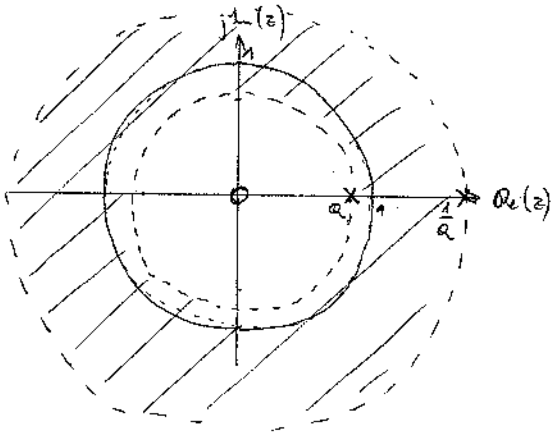
A 5.8

$$h[n] = h_1[n] + h_1[-n] = a^n \sigma[n] - \frac{1}{2}\delta[n] + a^{-n}\sigma[-n] - \frac{1}{2}\delta[n] = a^{-n}\sigma[-n] - \delta[n] + a^n\sigma[n]$$

(a) $$h[n] \circ\!\!-\!\!\bullet\, H(z) = \frac{z^{-1}}{z^{-1}-a} - 1 + \frac{z}{z-a} = \frac{1}{1-az} - 1 + \frac{z}{z-a} = -\frac{1}{a}\,\frac{1}{z-\frac{1}{a}} - 1 + \frac{z}{z-a} =$$

$$= \frac{\cancel{1} - \frac{z}{a} - \cancel{z^2} + \left(a+\frac{1}{a}\right)z - \cancel{1} + \cancel{z^2} - \frac{z}{a}}{\left(z-\frac{1}{a}\right)\left(z-a\right)} = \frac{z\left[\left(a+\frac{1}{a}\right) - \frac{2}{a}\right]}{\left(z-\frac{1}{a}\right)\left(z-a\right)} = \frac{z\left(a-\frac{1}{a}\right)}{\left(z-\frac{1}{a}\right)\left(z-a\right)}$$

(b) $$H(z) = \left(a-\frac{1}{a}\right)\cdot\frac{z}{\left(z-\frac{1}{a}\right)\left(z-a\right)}$$




$z_{01} = 0$ $\qquad z_{\infty 1} = a$ $\quad$ (rs)

$z_{02} = \infty$ $\qquad z_{\infty 2} = \frac{1}{a}$ $\quad$ (ls)

$|a| < 1$: $\quad a < R < \frac{1}{a}$

$|a| \geq 1$: kein Konvergenzband

Filter stabil für $|a| < 1$

(c) $$H(e^{j\Theta}) = \left(a-\frac{1}{a}\right)\frac{e^{j\Theta}}{\left(e^{j\Theta}-\frac{1}{a}\right)\left(e^{j\Theta}-a\right)} = \frac{e^{j\Theta}\left(a^2-1\right)}{a\cdot\left[e^{j2\Theta} - \left(a+\frac{1}{a}\right)e^{j\Theta} + 1\right]} = \frac{a^2-1}{a\cdot\left(e^{j\Theta} - \left[a+\frac{1}{a}\right] + e^{-j\Theta}\right)} = \frac{a^2-1}{a\cdot\left[2\cos(\Theta) - \left(a+\frac{1}{a}\right)\right]} =$$

$$= \frac{a^2-1}{2a\cos(\Theta) - \left(a^2+1\right)} = \frac{a^2-1}{-\left(a^2+1\right)}\cdot\frac{1}{1-\frac{2a}{a^2+1}\cos(\Theta)} = \frac{1-a^2}{1+a^2}\cdot\frac{1}{1-\frac{2a}{a^2+1}\cos(\Theta)}$$

$$|H(e^{j\Theta})| = \frac{1}{1+a^2}\cdot\left|\frac{1-a^2}{1-\frac{2a}{a^2+1}\cos(\Theta)}\right|$$

$$\mathrm{arc}\left\{H(e^{j\Theta})\right\} = 0$$

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(d) $a[n] = (h * \sigma)[n]$

$h[n] = h_1[n] + h_2[n] + h_3[n] = a^{-n}\sigma[-n] - \delta[n] + a^n \sigma[n]$

$a[n] = a_1[n] + a_2[n] + a_3[n]$

$a_2[n] = -\sigma[n]$

$a_3[n] = \sum_{k=-\infty}^{\infty} a^k \sigma[k]\sigma[n-k] = \sigma[n] \sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a}\sigma[n]$

$a_1[n] = \sum_{k=-\infty}^{\infty} a^{-k}\sigma[-k]\sigma[n-k] = \sum_{m=-\infty}^{\infty} a^m \sigma[m]\sigma[m+n] = \sum_{k=-\infty}^{\infty} a^k \sigma[k]\sigma[k+n]$

$n \geq 0: \quad a_1[n] = \sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$

$n < 0: \quad a_1[n] = \sum_{k=0}^{\infty} a^k - \sum_{k=0}^{-n-1} a^k = \frac{1}{1-a} - \frac{1-a^{-n}}{1-a} = \frac{a^{-n}}{1-a}$

$$a[n] = \begin{cases} \dfrac{a^{-n}}{1-a} & n < 0 \\[2ex] \dfrac{1+a-a^{n+1}}{1-a} & n \geq 0 \end{cases}$$