A 6.3 $\quad X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi k}{N} n} \qquad x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{j\frac{2\pi n}{N}k}$

(a) $X[0] = \sum_{n=0}^{N-1} x[n] = x[0] + x[1] + x[2] + \dots + x[N-3] + x[N-2] + x[N-1] =$

$x[n] = -x[N-1-n]$

$$= x[0] + x[1] + x[2] + \dots - x[2] - x[1] - x[0] = \begin{cases} 0 & N \in \mathbb{N}_g \\ x[\frac{N-1}{2}] & N \in \mathbb{N}_u = 0 \text{ laut Angabe} \end{cases}$$

(b) $X[\frac{N}{2}] = \sum_{n=0}^{N-1} x[n] \cdot (-1)^k = x[0] - x[1] + x[2] - x[3] + - \dots + x[N-4] - x[N-3] + x[N-2] - x[N-1] =$

$x[n] = x[N-1-n]$

$$= x[0] - x[1] + x[2] - x[3] + - \dots + x[3] - x[2] + x[1] - x[0] = 0$$

(c) $\sum_{n=0}^{N-1} |x[n]|^2 = \sum_{n=0}^{N-1} x[n]\, x^*[n] = \sum_{n=0}^{N-1} \left\{ \frac{1}{N}\left[\sum_{k=0}^{N-1} X[k]\, e^{j\frac{2\pi n}{N}k}\right] \frac{1}{N}\left[\sum_{l=0}^{N-1} X^*[l]\, e^{-j\frac{2\pi n}{N}l}\right]\right\} =$

$$= \frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} X[k]X^*[l] \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}(k-l)n} = \frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} X[k]X^*[l]\, \frac{1-e^{j2\pi(k-l)}}{1-e^{j\frac{2\pi}{N}(k-l)}} =$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X[k]X^*[k] = \frac{1}{N}\sum_{k=0}^{N-1} |X[k]|^2$$