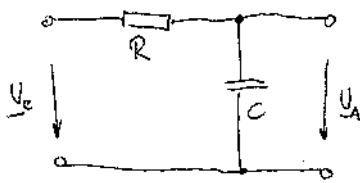


Tiefpassfilter

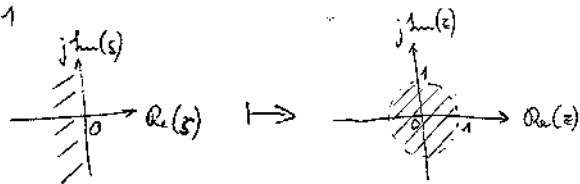


$$\frac{U_a}{U_e} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega RC + 1} \quad ; \quad RC = \frac{1}{\omega_g}$$

$$H_e(s) = \frac{1}{s+1}$$

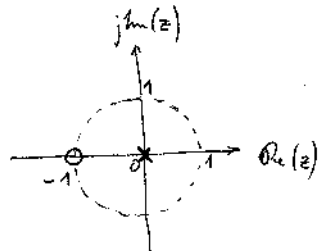
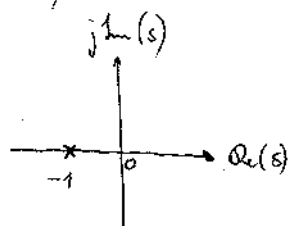
$$H_{TP}(z) = H_{e,TP}(s) \Big|_{s = \frac{1}{T} \frac{z-1}{z+1}}$$

$$v = \tan\left(\pi \frac{f_g}{f_s}\right)$$

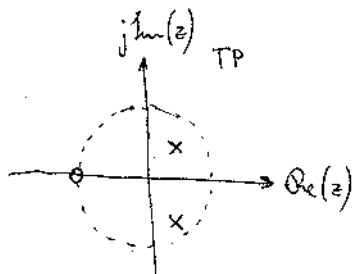
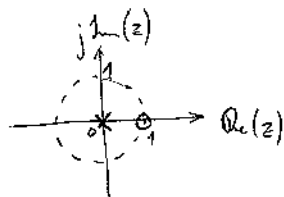


$$v=1: \quad H_{TP}(z) = \frac{1}{\frac{z-1}{z+1} + 1} = \frac{z+1}{2z}$$

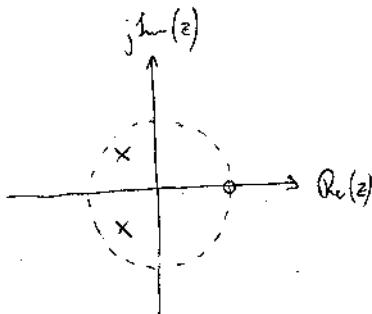
$(f_g = \frac{f_s}{4})$



$$(a) \quad H_{TP}(-z) = \frac{-z+1}{-2z} = \frac{z-1}{2z}$$



\mapsto



$$\frac{z+1}{z^2 - z + \frac{1}{2}}$$

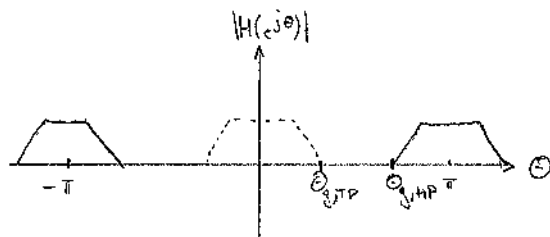
$$\mapsto (-1) \frac{z-1}{z^2 + z + \frac{1}{2}}$$

$$(c) \quad z = e^{j\theta}$$

$$-z = -e^{j\theta} = e^{j\pi} e^{j\theta} = e^{-j\pi} e^{j\theta} = e^{j(\theta \pm \pi)}$$

$$\theta_{g,HP} = \pi - \theta_{g,TP}$$

$$f_{g,HP} = \frac{f_s}{2} - f_{g,TP}$$

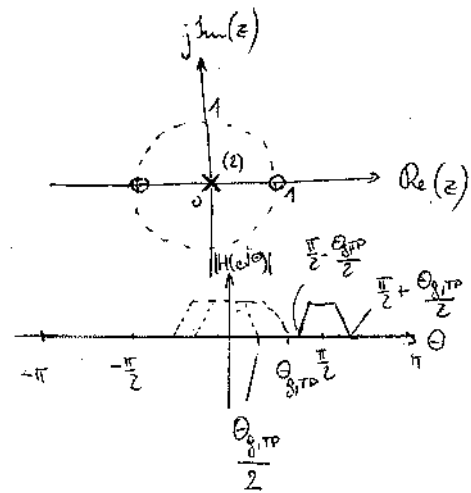


(c) nein

(d) siehe (a) - (c)

$$(e) \quad H_{BP}(z) = \frac{-z^2 + 1}{-2z^2} = \frac{z^2 - 1}{2z^2} = \frac{(z+1)(z-1)}{2z^2}$$

$$-z^2 = -e^{j2\theta} = e^{j\pi} e^{j2\theta} = e^{-j\pi} e^{j2\theta} = e^{j2\theta \pm \pi}$$



$$\Theta_{g1, BP} = \frac{\pi}{2} - \frac{\Theta_{g, TP}}{2}$$

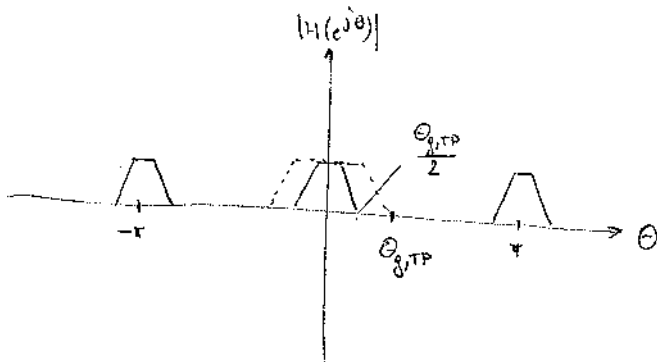
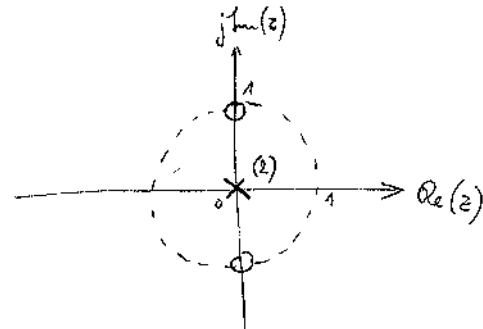
$$\Theta_{g2, BP} = \frac{\pi}{2} + \frac{\Theta_{g, TP}}{2}$$

$$\varphi_{g1, BP} = \frac{\varphi_s}{4} - \frac{\varphi_{g, TP}}{2}$$

$$\varphi_{g2, BP} = \frac{\varphi_s}{4} + \frac{\varphi_{g, TP}}{2}$$

$$H_{BS}(z) = \frac{z^2 + 1}{2z^2} = \frac{(z+j)(z-j)}{2z^2}$$

$$z^2 = e^{j2\theta}$$



$$\Theta_{g1, BS} = \frac{\Theta_{g, TP}}{2}$$

$$\varphi_{g1, BS} = \frac{\varphi_{g, TP}}{2}$$

$$\Theta_{g2, BS} = \pi - \frac{\Theta_{g, TP}}{2}$$

$$\varphi_{g2, BS} = \frac{\varphi_s}{2} - \frac{\varphi_{g, TP}}{2}$$