A4.7

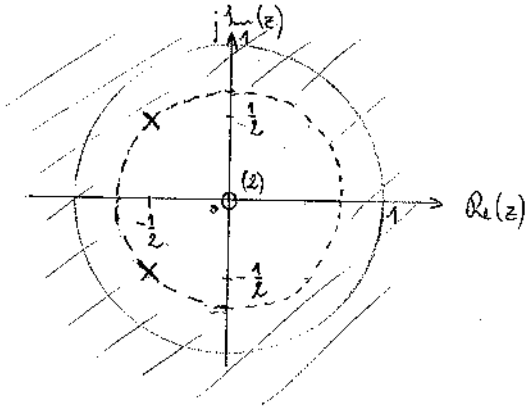
(a) $Y(z) + z^{-1}Y(z) + \frac{1}{2}z^{-2}Y(z) = \frac{5}{2}X(z)$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{5}{2}\cdot\left\{1 + z^{-1} + \frac{1}{2}z^{-2}\right\}^{-1} = \frac{5}{2}\,\frac{2z^2}{2z^2+2z+1} = \frac{5}{2}\,\frac{z^2}{z^2+z+\frac{1}{2}}$$

(b)

$z_{01,2} = 0$

$z_{\infty 1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4}-\frac{1}{2}} = -\frac{1}{2} \pm j\frac{1}{2}$



(c) $$A(z) = \frac{5}{2}\frac{z^2}{z^2+z+\frac{1}{2}}\cdot\frac{z}{z-1} = \frac{5}{2}\cdot\left\{1 + \frac{1}{2}\frac{z+1}{\left(z^2+z+\frac{1}{2}\right)(z-1)}\right\} = \frac{5}{2} + \frac{5}{4}\cdot\frac{z+1}{\left(z^2+z+\frac{1}{2}\right)(z-1)} =$$

$z^3 : z^3 - \frac{1}{2}z - \frac{1}{2} = 1 + \frac{1}{2}\frac{z+1}{z^3+\frac{1}{2}z-\frac{1}{2}}$

$$= \frac{5}{2} + \frac{5}{4}\cdot\left\{\frac{\alpha}{z-1} + \frac{\beta z+\gamma}{\left(z+\frac{1}{2}\right)^2+\frac{1}{4}}\right\}$$

$\alpha = \frac{4}{5}$

$z=0: \quad -2 = -\frac{4}{5} + 2\gamma \Rightarrow \gamma = -\frac{3}{5}$

$z=-\frac{1}{2}: \quad -\frac{4}{5}\cdot\frac{2}{3} + \frac{-\frac{1}{2}\beta - \frac{3}{5}}{\frac{1}{4}} = -\frac{4}{3} \Rightarrow \beta = -\frac{4}{5}$

$$A(z) = \frac{5}{2} + z^{-1}\frac{z}{z-1} + \frac{-z}{z^2+z+\frac{1}{2}} + \frac{3}{4}z^{-1}\frac{-z}{z^2+z+\frac{1}{2}}$$

$\rho = \frac{\sqrt{2}}{2}$ $\qquad \rho = \frac{\sqrt{2}}{2}$

$\sqrt{2}\cos(\alpha) = -1 \Rightarrow \alpha = \frac{3\pi}{4}$

$\sin(\alpha) = \frac{\sqrt{2}}{2} \Rightarrow k = -2$

$$A(z) \bullet\!\!-\!\!\circ\; a[n] = \frac{5}{2}\delta[n] + \sigma[n-1] - 2\cdot\left(\frac{\sqrt{2}}{2}\right)^n \sin\left(\frac{3\pi}{4}n\right)\sigma[n] - \frac{3}{2}\left(\frac{\sqrt{2}}{2}\right)^{n-1}\sin\left(\frac{3\pi}{4}(n-1)\right)\sigma[n-1]$$

$$A(z) = \frac{5}{2}\delta[n] + \sigma[n-1] - 2\left(\frac{\sqrt{2}}{2}\right)^n \sin\left(\frac{3\pi}{4}n\right)\sigma[n] + \frac{3}{2}\left(\frac{\sqrt{2}}{2}\right)^n\left[\sin\left(\frac{3\pi}{4}n\right) + \cos\left(\frac{3\pi}{4}n\right)\right]\sigma[n] - \frac{3}{2}\delta[n] =$$

$$= \sigma[n] - \frac{1}{2}\cdot\left(\frac{\sqrt{2}}{2}\right)^n \sin\left(\frac{3\pi}{4}n\right)\sigma[n] + \frac{3}{2}\left(\frac{\sqrt{2}}{2}\right)^n \cos\left(\frac{3\pi}{4}n\right)\sigma[n]$$

(d) $$Y(z) + z^{-1}\left(Y(z) + y[-1]\cdot z\right) + \frac{1}{2}z^{-2}\left(Y(z) + y[-1]z + y[-2]z^2\right) = \frac{5}{2}X(z)$$

$$Y(z)\cdot\left\{1 + z^{-1} + \frac{1}{2}z^{-2}\right\} + y[-1]\cdot\left(1 + \frac{1}{2}z^{-1}\right) + y[-2]\cdot\frac{1}{2} = \frac{5}{2}X(z)$$

$$X(z) = Y(z) = \frac{z}{z-1}$$

$$y[-1]\cdot\frac{z+\frac{1}{2}}{z} + y[-2]\cdot\frac{1}{2} = \frac{5}{2}\frac{z}{z-1} - \frac{z^2+z+\frac{1}{2}}{z^2}\cdot\frac{z}{z-1}$$

$$y[-1]\cdot\left(z+\frac{1}{2}\right)(z-1)z + y[-2]\cdot\frac{1}{2}\cdot z^2(z-1) = \frac{5}{2}z^3 - z^3 - z^2 - \frac{1}{2}z$$

$$\left(y[-1] + \frac{1}{2}y[-2]\right)z^3 - \left(\frac{1}{2}y[-1] + \frac{1}{2}y[-2]\right)z^2 - \frac{1}{2}y[-1]\cdot z = \frac{3}{2}z^3 - z^2 - \frac{1}{2}z$$

$\Rightarrow y[-1] = 1$

$\Rightarrow y[-2] = 1$