A 4.2

(a) $\frac{1}{1-\frac{1}{2}z^{-1}} = \frac{z}{z-\frac{1}{2}}$

$x_R[n] = \left(\frac{1}{2}\right)^n \sigma[n]$

$x_L[n] = -\left(\frac{1}{2}\right)^n \sigma[-n-1]$

(b) $\frac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-z^{-1}\right)} = \frac{z}{z-\frac{1}{2}} \cdot \frac{z}{z-1} = \frac{z^2}{z^2-\frac{3}{2}z+\frac{1}{2}} \overset{\text{PD/BZ}}{=} 1 - \frac{1}{2}\,\frac{1}{z-\frac{1}{2}} + 2\,\frac{1}{z-1}$

$z^2 : \left(z^2-\frac{3}{2}z+\frac{1}{2}\right) = 1 + \frac{\frac{3}{2}z-\frac{1}{2}}{z^2-\frac{3}{2}z+\frac{1}{2}} = 1 + \frac{3}{2}\cdot\frac{z-\frac{1}{3}}{\left(z-\frac{1}{2}\right)(z-1)}$

$\frac{z-\frac{1}{3}}{\left(z-\frac{1}{2}\right)(z-1)} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-1}$

$A = \frac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}-1} = -\frac{1}{3}$

$B = \frac{1-\frac{1}{3}}{1-\frac{1}{2}} = \frac{4}{3}$

$X(z) = 1 - \frac{1}{2}z^{-1}\,\frac{z}{z-\frac{1}{2}} + 2z^{-1}\,\frac{z}{z-1}$

$x_R[n] = \delta[n] - \frac{1}{2}\cdot\left(\frac{1}{2}\right)^{n-1}\sigma[n-1] + 2\,\sigma[n-1] = \delta[n] - \left\{\left(\frac{1}{2}\right)^n - 2\right\}\sigma[n-1] + \left\{\left(\frac{1}{2}\right)^n + 2\right\}\sigma[n]$

$x_L[n] = \left\{\left(\frac{1}{2}\right)^n - 2\right\}\sigma[-n-1]$

A 4.2

(c) $$\frac{1-\frac{1}{2}z^{-1}}{1+\frac{1}{2}z^{-1}} = \frac{z-\frac{1}{2}}{z+\frac{1}{2}} = \frac{z}{z+\frac{1}{2}} - \frac{1}{2}z^{-1}\frac{z}{z+\frac{1}{2}}$$

$$x_A[n] = \left(\frac{1}{2}\right)^n \sigma[n] - \frac{1}{2}\left(\frac{1}{2}\right)^{n-1}\sigma[n-1] = \delta[n] + 2\left(-\frac{1}{2}\right)^n \sigma[n-1]$$

$$x_L[n] = -\left(\frac{1}{2}\right)^n \sigma[-n-1] + \frac{1}{2}\left(-\frac{1}{2}\right)^{n-1}\sigma[-n] = -\delta[n] - 2\left(-\frac{1}{2}\right)^n \sigma[-n-1]$$