A 3.1

(a) $\alpha^n \sin(\Theta_0 n)\,\sigma[n] = \frac{1}{2j}\,\alpha^n \sigma[n]\cdot\left(e^{j\Theta_0 n} - e^{-j\Theta_0 n}\right)$

$\alpha^n \sigma[n] \circ\!\!-\!\!\bullet \frac{1}{1-\alpha e^{-j\Theta}}$; $|\alpha| < 1$

$$X(e^{j\Theta}) = \frac{1}{2j}\cdot\left\{\frac{1}{1-\alpha e^{-j(\Theta-\Theta_0)}} - \frac{1}{1-\alpha e^{-j(\Theta+\Theta_0)}}\right\} =$$

$$= \frac{1}{2j}\;\frac{1-\alpha e^{-j(\Theta+\Theta_0)} - 1 + \alpha e^{-j(\Theta-\Theta_0)}}{1+\alpha^2 e^{-j2\Theta} - \alpha\left(e^{-j(\Theta-\Theta_0)} + e^{-j(\Theta+\Theta_0)}\right)} =$$

$$= \frac{1}{2j}\;\frac{\alpha e^{-j\Theta}\left(e^{j\Theta_0} - e^{-j\Theta_0}\right)}{1+\alpha^2 e^{-j2\Theta} - \alpha e^{-j\Theta}\left(e^{j\Theta_0} + e^{-j\Theta_0}\right)} = \frac{\alpha e^{-j\Theta}\sin(\Theta_0)}{1+\alpha^2 e^{-j2\Theta} - 2\alpha e^{-j\Theta}\cos(\Theta_0)}$$

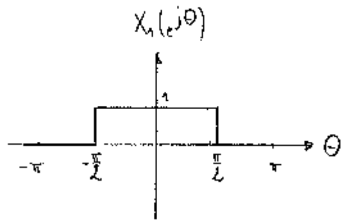
(b) $2^n \sigma[-n] = \left(\frac{1}{2}\right)^{-n}\sigma[-n]$

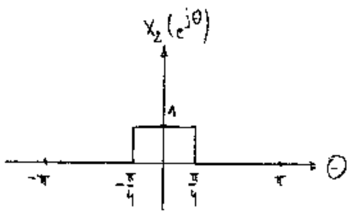
$\left(\frac{1}{2}\right)^n \sigma[n] \circ\!\!-\!\!\bullet \frac{1}{1-\frac{1}{2}e^{-j\Theta}}$

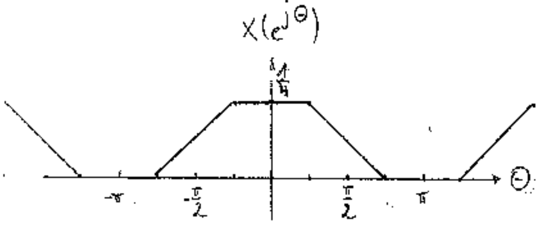
$x[-n] \circ\!\!-\!\!\bullet X(e^{-j\Theta})$

$$X(e^{j\Theta}) = \frac{1}{1-\frac{1}{2}e^{j\Theta}}$$

(c) $x_1[n] = \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}$ $\qquad$ $x_2[n] = \frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}$







$$X(e^{j\Theta}) = \frac{1}{2\pi}(X_1 * X_2)(e^{j\Theta}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\vartheta})\,X_2(e^{j(\Theta-\vartheta)})\,d\vartheta$$

A 3.1

(d) $\left(\frac{1}{2}\right)^{|n|} = \left(\frac{1}{2}\right)^{-n} \sigma[-n] + \left(\frac{1}{2}\right)^{n} \sigma[n] - \delta[n]$

$$X(e^{j\theta}) = \frac{1}{1-\frac{1}{2}e^{j\theta}} + \frac{1}{1-\frac{1}{2}e^{j\theta}} - 1 = \frac{1-\frac{1}{2}e^{j\theta} + 1-\frac{1}{2}e^{-j\theta} - \left[1+\frac{1}{4}-\frac{1}{2}\left(e^{j\theta}+e^{-j\theta}\right)\right]}{1+\frac{1}{4}-\frac{1}{2}\left(e^{j\theta}+e^{-j\theta}\right)} =$$

$$= \frac{\frac{3}{4} - 0\cdot\cos(\theta)}{\frac{5}{4} - \cos(\theta)} = \frac{3}{5-4\cos(\theta)}$$

(e) $n\,x[n] \circ\!\!-\!\!\bullet\ j\,\frac{dX(e^{j\theta})}{d\theta}$

$$\frac{d}{d\theta}\,\frac{3}{5-4\cos(\theta)} = \frac{-12\sin(\theta)}{\left[5-4\cos(\theta)\right]^2}$$

$$X(e^{j\theta}) = -\frac{12j\sin(\theta)}{\left[5-4\cos(\theta)\right]^2}$$

(f) $(-1)^n = e^{-j\pi n} = \cos(\pi n) - j\sin(\pi n) = \cos(\pi n) \qquad (n \in \mathbb{Z}\,!)$

$$X(e^{j\theta}) = \pi\left\{\delta_{2\pi}(\theta-\pi) + \delta_{2\pi}(\theta+\pi)\right\} = 2\pi\,\delta_{2\pi}(\theta-\pi)$$

$$\delta_{2\pi}(\theta-\pi) = \delta_{2\pi}(\theta+\pi) = \sum_{k=-\infty}^{\infty}\delta(\theta-\pi-2\pi k)$$

A 3.1

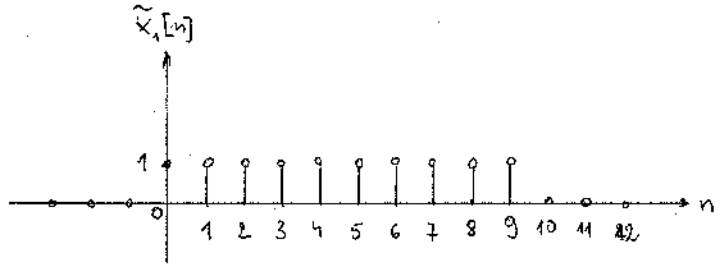
(g) $x[n] =: x_1[n+9]; \; (\tilde{x}_1 * \tilde{x}_1)[n] \cdot \frac{1}{10} = x_1[n]$

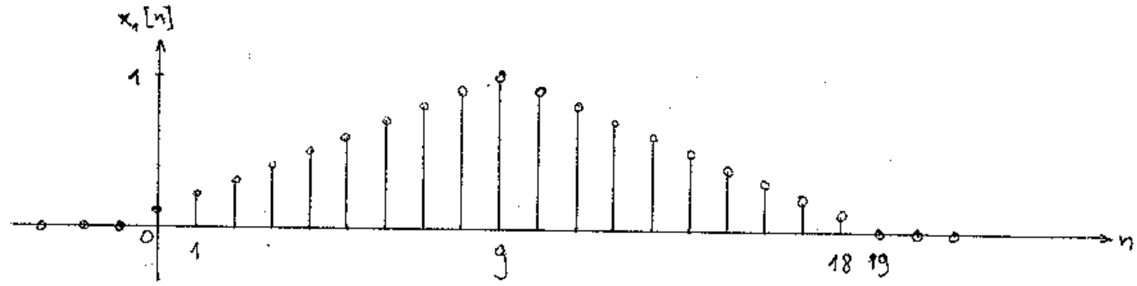
$$\tilde{x}_1[n] = \begin{cases} 1 & 0 \le n \le 9 \\ 0 & \text{sonst} \end{cases}$$

$$\tilde{x}_1[n] \circ\!\!-\!\!\bullet \tilde{X}_1(e^{j\theta}) = \sum_{n=0}^{9} e^{-j\theta n} = \frac{1-e^{-j10\theta}}{1-e^{-j\theta}} = \frac{e^{-j5\theta}}{e^{-j\frac{\theta}{2}}} \frac{e^{j5\theta}-e^{-j5\theta}}{e^{j\frac{\theta}{2}}-e^{-j\frac{\theta}{2}}} = e^{-j\frac{9}{2}\theta} \frac{\sin(5\theta)}{\sin\left(\frac{\theta}{2}\right)}$$

$$\tilde{X}_1(e^{j\theta}) \cdot \tilde{X}_1(e^{j\theta}) = \tilde{X}_1^2(e^{j\theta}) = 10 \cdot X_1(e^{j\theta}) = e^{-j9\theta} \frac{\sin^2(5\theta)}{\sin^2\left(\frac{\theta}{2}\right)}$$

$$X(e^{j\theta}) = e^{j9\theta} X_1(e^{j\theta}) = \frac{1}{10} \frac{\sin^2(5\theta)}{\sin^2\left(\frac{\theta}{2}\right)}$$





$$x_1[n] = \frac{1}{10} \cdot \sum_{k=-\infty}^{\infty} \tilde{x}_1[k]\, \tilde{x}_1[n-k]$$