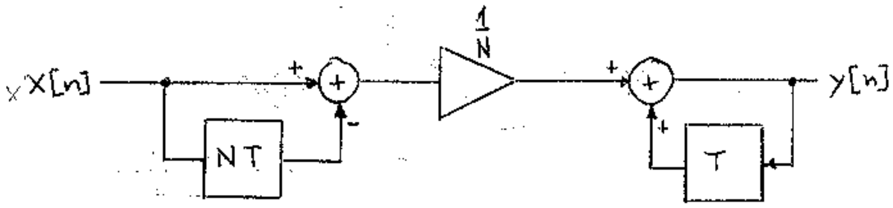
A 2.7

$$y[n] = y[n-1] + \frac{1}{N}\Big(x[n] - x[n-N]\Big) \quad ; \quad N \geq 0$$

(a)



(b) über Fourier-Transformation

$$h[n] \circ\!\!-\!\!\bullet\; H(e^{j\Theta}) = \frac{Y(e^{j\Theta})}{X(e^{j\Theta})}$$

$$y[n] = y[n-1] + \frac{1}{N}\Big(x[n] - x[n-N]\Big)$$

$$Y(e^{j\Theta}) = e^{-j\Theta}\,Y(e^{j\Theta}) + \frac{1}{N}\Big\{X(e^{j\Theta}) - e^{-j\Theta N}X(e^{j\Theta})\Big\}$$

$$Y(e^{j\Theta})\Big\{1 - e^{-j\Theta}\Big\} = X(e^{j\Theta})\cdot\frac{1}{N}\Big\{1 - e^{-j\Theta N}\Big\}$$

$$H(e^{j\Theta}) = \frac{1}{N}\,\frac{1-e^{-j\Theta N}}{1-e^{-j\Theta}} = \frac{1}{N}\left\{\frac{1}{1-e^{-j\Theta}} - e^{-j\Theta N}\frac{1}{1-e^{-j\Theta}}\right\}$$

$$h[n] = \frac{1}{N}\Big(\sigma[n] - \sigma[n-N]\Big)$$

$$a[n] = (h * \sigma)[n] = \sum_{k=-\infty}^{\infty}\frac{1}{N}\Big(\sigma[k]-\sigma[k-N]\Big)\sigma[n-k] =$$

$$= \frac{1}{N}\left\{\sigma[n]\sum_{k=0}^{n}1 - \sigma[n-N]\sum_{k=N}^{n}1\right\} = \frac{1}{N}\Big\{\sigma[n]\,(n+1) - \sigma[n-N]\,(n+1) + \sigma[n-N]\cdot N\Big\} =$$

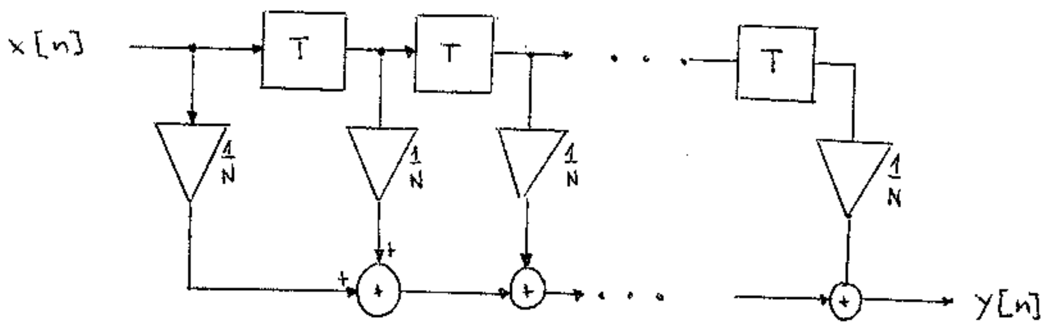
$$= \frac{1}{N}\cdot\Big\{\big(\sigma[n]-\sigma[n-N]\big)(n+1) + N\,\sigma[n-N]\Big\}$$

A 2.7

(c) $h[n] = y[n] \Big|_{x[n] = \delta[n]}$

$$h[n] = \frac{1}{N}\left(\sigma[n] - \sigma[n-N]\right) = \frac{1}{N}\sum_{k=0}^{N-1} \delta[n-k]$$

$$y[n] = \frac{1}{N}\sum_{k=0}^{N-1} x[n-k] = \frac{1}{N}\left\{ x[n] + x[n-1] + \cdots + x[n-(N-2)] + x[n-(N-1)] \right\}$$



System ohne Rückkopplung besteht aus: N Multiplizierern

N-1 Verzögerungselemente / Speicherelemente

N-1 Addierer