A 4.8

(a) $h[n] = \alpha^n \cdot \left(\sigma[n] - \sigma[n-N]\right) = \alpha^n \sigma[n] - \alpha^N \alpha^{n-N} \sigma[n-N]$

$$H(z) = \frac{z}{z-\alpha} - \alpha^N z^{-N} \frac{z}{z-\alpha} = \frac{z - \alpha^N z^{-N+1}}{z-\alpha} = \frac{z \cdot \left(1 - \alpha^N z^{-N}\right)}{z - \alpha} = \frac{z^N - \alpha^N}{(z-\alpha)\, z^{N-1}}$$

z_0 : $z^N - \alpha^N = 0$

$r^N e^{j\Theta N} = \alpha^N$

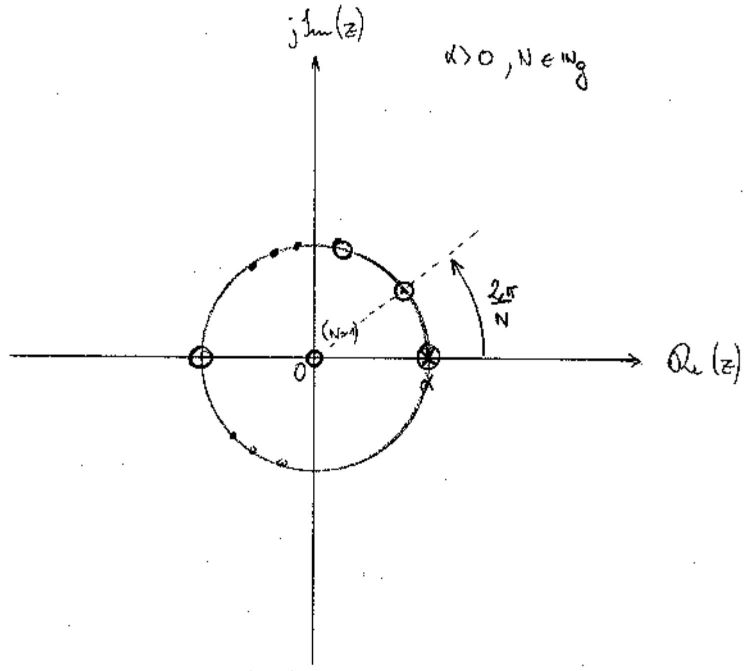
$r = \alpha$

$\Theta \cdot N = 2\pi \cdot k$; $\Theta = \frac{2\pi}{N} \cdot k$; $k = 0, \dots, N-1$

$z_{0\,1,\dots,N-1} = \alpha \cdot e^{j\frac{2\pi}{N}\cdot k}$; $k = 0, \dots, N-1$

$z_{\infty 1} = \alpha$

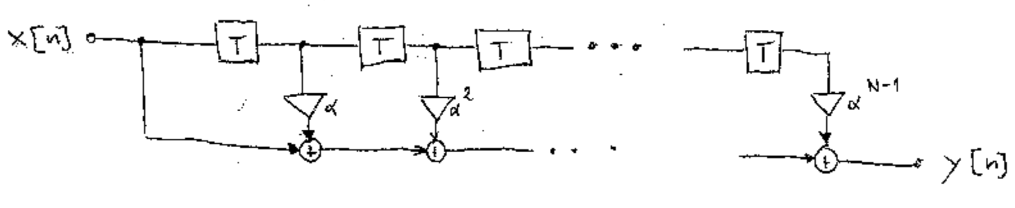
$z_{\infty 2,\dots,N} = 0$



Pol-/Nst-Kompensation tritt f. e. reellen α auf

$\Rightarrow$ stabil $\forall\, \alpha \in \mathbb{R}$

(b) $h[n] = \sum_{k=0}^{N-1} \alpha^k \delta[n-k] \quad \Rightarrow \quad y[n] = \sum_{k=0}^{N-1} \alpha^k x[n-k]$



(c) N-1 Addierer, Multiplizierer, Speicherelemente

(d)

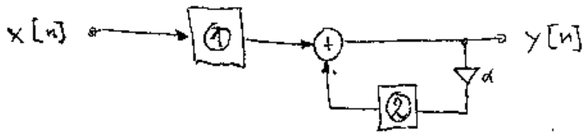
$$y[n]=\sum_{k=0}^{N-1}\alpha^k x[n-k] \;\circ\!\!-\!\!\bullet\; Y(z)=\sum_{k=0}^{N-1}\alpha^k z^{-k}X(z)=$$

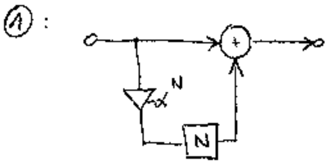
$$=X(z)\sum_{k=0}^{N-1}\left(\frac{\alpha}{z}\right)^k = X(z)\cdot\frac{1-\alpha^N z^{-N}}{1-\alpha z^{-1}}$$

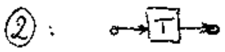
$$Y(z)-\alpha z^{-1}Y(z) = X(z)-\alpha^N z^{-N}X(z)$$

$$y[n]-\alpha y[n-1] = x[n]-\alpha^N x[n-N]$$

$$y[n] = x[n]-\alpha^N x[n-N]+\alpha y[n-1]$$







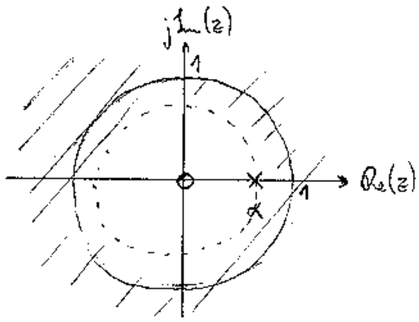
(e)

$$y[n]=x[n]+\alpha y[n-1]$$

$$Y(z)=X(z)+z^{-1}\alpha Y(z)$$

$$H(z)=\frac{1}{1-\alpha z^{-1}}=\frac{z}{z-\alpha}$$

$|\alpha|<1 \Rightarrow$ rekurs. Filter stabil



(f)

	(b)	(d)
ADD	N−1	2
MLT	N−1	2
SPE	N−1	N+1