A 2.2

$$h[n] = \alpha^n \sigma[n] \;;\; y[n] = (x * h)[n] = (h * x)[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]\, h[k]$$

(a) $$y[n] = \sum_{k=-\infty}^{\infty} \alpha^k \sigma[k] \cdot \sigma[n-k] = \sigma[n] \sum_{k=0}^{n} \alpha^k = \frac{1-\alpha^{n+1}}{1-\alpha} \sigma[n]$$

(b) $$y[n] = \sum_{k=-\infty}^{\infty} \alpha^k \sigma[k]\, \sigma[k-n] \quad ; \quad |\alpha| < 1$$

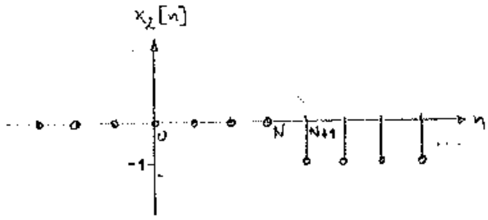
$$n \leq 0: \quad y[n] = \sum_{k=0}^{\infty} \alpha^k = \frac{1}{1-\alpha}$$

$$n > 0: \quad y[n] = \sum_{k=0}^{\infty} \alpha^k - \sum_{k=0}^{n-1} \alpha^k = \frac{1}{1-\alpha} - \frac{1-\alpha^n}{1-\alpha} = \frac{\alpha^n}{1-\alpha}$$

(c) $$x[n] = x_1[n] + x_2[n]$$

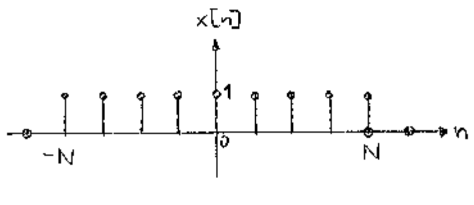
$x_1[n] = \sigma[n]$ (Antwort d. Systems siehe Punkt (a))

$x_2[n] = \sigma[-n+N] - 1 = -x_1[n-(N+1)]$ (Zeitverschiebung, Faktor (-1))



$$y[n] = y_1[n] + y_2[n] = y_1[n] - y_1[n-(N+1)] = \frac{1-\alpha^{n+1}}{1-\alpha} \sigma[n] - \frac{1-\alpha^{n-N}}{1-\alpha} \sigma[n-(N+1)]$$

(d) $$x[n] = \sigma[n+N] \cdot \sigma[-n+N] = \sigma[n+N] - \sigma[n-(N+1)] \qquad (N \geq 0)$$



$$y[n] = \frac{1-\alpha^{n+N+1}}{1-\alpha} \sigma[n+N] - \frac{1-\alpha^{n-N}}{1-\alpha} \sigma[n-(N+1)]$$

A 2.2

(e) $$y[n] = \sum_{k=-\infty}^{\infty} \alpha^k \sigma[k] \beta^{n-k} \sigma[n-k] = \sigma[n] \sum_{k=0}^{n} \alpha^k \beta^{n-k} = \beta^n \sigma[n] \sum_{k=0}^{n} \left(\frac{\alpha}{\beta}\right)^k =$$

$$= \beta^n \frac{1-\alpha^{n+1}\beta^{-n-1}}{1-\alpha\beta^{-1}} \sigma[n] = \left(\frac{\beta^n}{1-\frac{\alpha}{\beta}} - \frac{\frac{\alpha^{n+1}}{\beta}}{1-\frac{\alpha}{\beta}} \right) \sigma[n]$$

$$= \frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} \sigma[n]$$

$\alpha = \beta$: $$y[n] = \lim_{\beta \to \alpha} \frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} \sigma[n] = \lim_{\beta \to \alpha} (n+1) \beta^n \sigma[n] = (n+1) \alpha^n \sigma[n]$$

ODER:

$$y[n] = \alpha^n \sigma[n] \sum_{k=0}^{n} 1 = (n+1) \alpha^n \sigma[n]$$