

A 6.1  $x[n] \rightarrow X(e^{j\theta})$

$$y[n] = \sum_{m=-\infty}^{\infty} x[n+mN], \quad y[n+N] = y[n]$$

$$\begin{aligned} c_k &= \frac{1}{N} \sum_{n=0}^{N-1} y[n] e^{-j \frac{2\pi k}{N} n} = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{m=-\infty}^{\infty} x[n+mN] e^{-j \frac{2\pi k}{N} n} \\ &= \frac{1}{N} \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} x[n+mN] e^{-j \frac{2\pi k}{N} n} \end{aligned}$$

$$n+m \cdot N = l, \quad n = l - mN$$

$$\begin{aligned} &= \frac{1}{N} \sum_{m=-\infty}^{\infty} \sum_{l=mN}^{N-1+mN} x[l] e^{-j \frac{2\pi k}{N} l} \cdot \underbrace{e^{j \frac{2\pi k}{N} mN}}_1 = \frac{1}{N} \sum_{m=-\infty}^{\infty} \sum_{l=mN}^{N-1+mN} x[l] e^{-j \frac{2\pi k}{N} l} \\ &= \frac{1}{N} \sum_{l=-\infty}^{\infty} x[l] e^{-j \frac{2\pi k}{N} l} = \frac{1}{N} X(e^{j\theta}) \Big|_{\theta = \frac{2\pi k}{N}} = \frac{1}{N} X(e^{j \frac{2\pi k}{N}}) \end{aligned}$$

$$\left. \begin{aligned} c_k &= \frac{1}{N} \sum_{n=0}^{N-1} y[n] e^{-j \frac{2\pi k}{N} n} \\ Y[k] &= \sum_{n=0}^{N-1} y[n] e^{-j \frac{2\pi k}{N} n} \end{aligned} \right\} \Rightarrow c_k = \frac{1}{N} \cdot Y[k]$$