A 6.1 $x[n] \circ\!\!-\!\!\bullet X(e^{j\Theta})$

$$y[n] = \sum_{m=-\infty}^{\infty} x[n+mN] \quad , y[n+N] = y[n]$$

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} y[n]\, e^{-j\frac{2\pi k}{N}n} = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{m=-\infty}^{\infty} x[n+mN]\, e^{-j\frac{2\pi k}{N}\cdot n} =$$

$$= \frac{1}{N}\sum_{m=-\infty}^{\infty}\sum_{n=0}^{N-1} x[n+mN]\, e^{-j\frac{2\pi k}{N}\cdot n}$$

$$n + m\cdot N = \ell \quad , n = \ell - mN$$

$$= \frac{1}{N}\sum_{m=-\infty}^{\infty}\sum_{\ell=mN}^{N-1+mN} x[\ell]\, e^{-j\frac{2\pi k}{N}\ell}\cdot \underbrace{e^{j2\pi k\cdot m}}_{1} = \frac{1}{N}\sum_{m=-\infty}^{\infty}\sum_{\ell=mN}^{N-1+mN} x[\ell]\, e^{-j\frac{2\pi k}{N}\ell} =$$

$$= \frac{1}{N}\sum_{\ell=-\infty}^{\infty} x[\ell]\, e^{-j\frac{2\pi k}{N}\cdot \ell} = \frac{1}{N} X(e^{j\Theta})\Big|_{\Theta=\frac{2\pi k}{N}} = \frac{1}{N} X\left(e^{j\frac{2\pi k}{N}}\right)$$

$$\left.\begin{aligned} c_k &= \frac{1}{N}\cdot\sum_{n=0}^{N-1} y[n]\, e^{-j\frac{2\pi k}{N}\cdot n} \\ Y[k] &= \sum_{n=0}^{N-1} y[n]\, e^{-j\frac{2\pi k}{N}\cdot n} \end{aligned}\right\} \Rightarrow c_k = \frac{1}{N}\cdot Y[k]$$