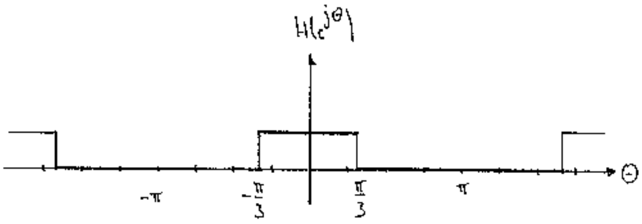
A 3.3

$$h[n] = \frac{\sin\left(\frac{\pi}{3}n\right)}{\pi n} \quad \circ\!\!-\!\!\bullet \quad H(e^{j\Theta}) = \mathrm{rect}_{2\pi}\left(\frac{\Theta}{\frac{2\pi}{3}}\right) \quad ; \quad \text{LTI}$$



$$y[n] = (x * h)[n] \quad \circ\!\!-\!\!\bullet \quad X(e^{j\Theta}) \cdot H(e^{j\Theta})$$

(b) $$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-8k] = \delta_8[n] \quad \circ\!\!-\!\!\bullet \quad \frac{2\pi}{8} \sum_{k=-\infty}^{\infty} \delta\left(\Theta - \frac{2\pi}{8}k\right) = \frac{\pi}{4}\, \delta_{\frac{\pi}{4}}(\Theta)$$

$$Y(e^{j\Theta}) = \frac{\pi}{4}\left\{ \delta_{2\pi}(\Theta) + \delta_{2\pi}\left(\Theta + \frac{\pi}{4}\right) + \delta_{2\pi}\left(\Theta - \frac{\pi}{4}\right)\right\} = \frac{1}{8}\, 2\pi\, \delta_{2\pi}(\Theta) + \frac{1}{4}\pi \left\{ \delta_{2\pi}\left(\Theta + \frac{\pi}{4}\right) + \delta_{2\pi}\left(\Theta - \frac{\pi}{4}\right)\right\}$$

$$y[n] = \frac{1}{8} + \frac{1}{4}\cos\left(\frac{\pi}{4}n\right)$$

(c) $$x[n] = \delta_8[n] + \delta_8[n+1] + \delta_8[n-1] + \delta_8[n+2] + \delta_8[n-2] = \sum_{\ell=-2}^{2} \delta_8[n-\ell]$$

$$\Rightarrow y[n] = \sum_{\ell=-2}^{2}\left\{\frac{1}{8} + \frac{1}{4}\cos\left(\frac{\pi}{4}(n-\ell)\right)\right\} = \frac{5}{8} + \frac{1}{4}\sum_{\ell=-2}^{2}\cos\left(\frac{\pi}{4}(n-\ell)\right)$$

$$\sum_{\ell=-2}^{2}\cos\left(\frac{\pi}{4}(n-\ell)\right) = \sum_{\ell=0}^{4}\cos\left(\frac{\pi}{4}(n-\ell+2)\right) = \frac{1}{2}\sum_{\ell=0}^{4}\left(e^{j\frac{\pi}{4}(n-\ell+2)} + e^{-j\frac{\pi}{4}(n-\ell+2)}\right) =$$

$$= \frac{1}{2}\cdot\left\{ e^{j\frac{\pi}{4}n} e^{j\frac{\pi}{2}} \sum_{\ell=0}^{4} e^{-j\frac{\pi}{4}\ell} + e^{-j\frac{\pi}{4}n} e^{-j\frac{\pi}{2}} \sum_{\ell=0}^{4} e^{j\frac{\pi}{4}\ell}\right\} = \frac{1}{2}\left\{ e^{j\frac{\pi}{4}n} e^{j\frac{\pi}{2}} \cdot \frac{1-e^{-j\frac{5\pi}{4}}}{1-e^{-j\frac{\pi}{4}}} + e^{-j\frac{\pi}{4}n} e^{-j\frac{\pi}{2}} \frac{1-e^{j\frac{5\pi}{4}}}{1-e^{j\frac{\pi}{4}}}\right\} =$$

$$\frac{1-e^{-j\frac{5\pi}{4}}}{1-e^{-j\frac{\pi}{4}}} = e^{-j\frac{\pi}{2}} \frac{\sin\left(\frac{5\pi}{8}\right)}{\sin\left(\frac{\pi}{8}\right)} \quad ; \quad \frac{1-e^{j\frac{5\pi}{4}}}{1-e^{j\frac{\pi}{4}}} = e^{j\frac{\pi}{2}} \frac{\sin\left(\frac{5\pi}{8}\right)}{\sin\left(\frac{\pi}{8}\right)}$$

$$= \cos\left(\frac{\pi}{4}n\right) \cdot \frac{\sin\left(\frac{5\pi}{8}\right)}{\sin\left(\frac{\pi}{8}\right)}$$

$$y[n] = \frac{5}{8} + \frac{1}{4}\frac{\sin\left(\frac{5\pi}{8}\right)}{\sin\left(\frac{\pi}{8}\right)} \cdot \cos\left(\frac{\pi}{4}n\right)$$

A 3.3

(c) $x[n] = \delta[n+1] + \delta[n-1]$

$$(x * h)[n] = \frac{\sin\left(\frac{\pi}{3}(n+1)\right)}{\pi(n+1)} + \frac{\sin\left(\frac{\pi}{3}(n-1)\right)}{\pi(n-1)}$$

(d) $x[n] = \dfrac{\sin\left(\frac{\pi}{4}n\right)}{\pi n} \circ\!\!-\!\!\bullet\; X(e^{j\theta}) = \mathrm{rect}_{2\pi}\left(\dfrac{\theta}{\frac{\pi}{2}}\right)$

$$X(e^{j\theta}) \cdot H(e^{j\theta}) = \mathrm{rect}_{2\pi}\left(\frac{\theta}{\frac{\pi}{2}}\right) \cdot \mathrm{rect}_{2\pi}\left(\frac{\theta}{\frac{2\pi}{3}}\right) = \mathrm{rect}_{2\pi}\left(\frac{\theta}{\frac{\pi}{2}}\right) = Y(e^{j\theta})$$

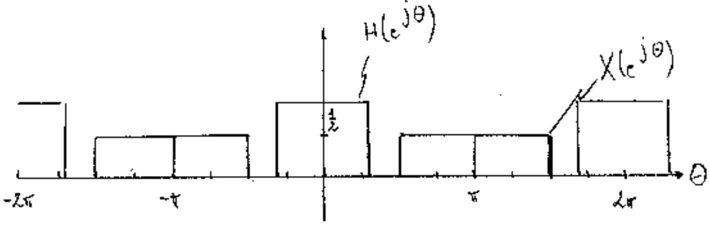
$y[n] = x[n]$

(e) $x[n] = \dfrac{\sin\left(\frac{\pi}{4}n\right)}{\pi n} \cos\left(\dfrac{3\pi}{4}n\right) = \dfrac{\sin\left(\frac{\pi}{4}n\right)}{\pi n} \cdot \dfrac{1}{2} \cdot \left(e^{j\frac{3\pi}{4}n} + e^{-j\frac{3\pi}{4}n}\right)$

ODER: $e^{-j\theta_0 n} x[n] \circ\!\!-\!\!\bullet\; X(e^{j(\theta-\theta_0)})$

$$X(e^{j\theta}) = \frac{1}{2} \cdot \frac{1}{\cancel{2\pi}} \mathrm{rect}_{2\pi}\left(\frac{\theta}{\frac{\pi}{2}}\right) * \left\{\cancel{2\pi}\, \delta_{2\pi}\left(\theta - \frac{3\pi}{4}\right) + \cancel{2\pi}\, \delta_{2\pi}\left(\theta + \frac{3\pi}{4}\right)\right\} =$$

$$= \frac{1}{2} \cdot \left\{ \mathrm{rect}_{2\pi}\left(\frac{\theta - \frac{3\pi}{4}}{\frac{\pi}{2}}\right) + \mathrm{rect}_{2\pi}\left(\frac{\theta + \frac{3\pi}{4}}{\frac{\pi}{2}}\right)\right\}$$



$$Y(e^{j\theta}) = X(e^{j\theta}) \cdot H(e^{j\theta}) = 0 \quad \bullet\!\!-\!\!\circ\; y[n] = 0$$