A 2.9

$$y[n] = \sum_{k=n-1}^{n+1} \left(x[k+1] - x[k] + x[k-1] \right) = x[n] - x[n-1] + x[n-2] + x[n+1] - x[n] + x[n-1] + x[n+2] - x[n+1] + x[n]$$

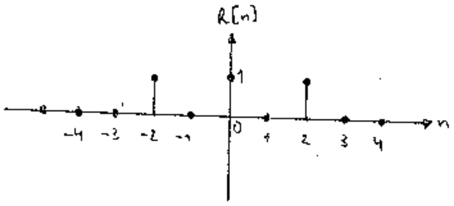
$$= x[n] + x[n-2] + x[n+2]$$

(a) (i) $x[n] = a x_1[n] + b x_2[n] \Rightarrow y[n] = a y_1[n] + b y_2[n]$

$$a x_1[n] + b x_2[n] + a x_1[n-2] + b x_2[n-2] + a x_1[n+2] + b x_2[n+2] =$$

$$= a \left(x_1[n] + x_1[n-2] + x_1[n+2] \right) + b \left(x_2[n] + x_2[n-2] + x_2[n+2] \right) \Rightarrow \text{linear}$$

(b) (ii) $h[n] = \delta[n+2] + \delta[n] + \delta[n-2] \neq 0$ f. $n < 0$ $\Rightarrow$ akausal



(iii) $|x[n]| \leq M < \infty \quad \forall n \Rightarrow |y[n]| \leq N < \infty$

$$\rightarrow |y[n]| = |x[n] + x[n-2] + x[n+2]| \leq |x[n]| + |x[n-2]| + |x[n+2]| \leq 3M < \infty$$

$\Rightarrow$ BIBO-stabil

(iv) $x[n-N_0] \Rightarrow y[n-N_0]$

$$y[n-N_0] = x[n-N_0] + x[n-N_0-2] + x[n-N_0+2]$$

$n - N_0 \mapsto m$

$$y[m] = x[m] + x[m-2] + x[m+2]$$

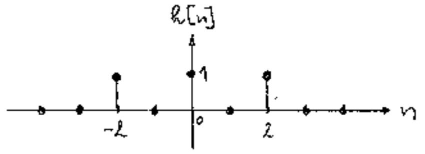
$m \mapsto n$

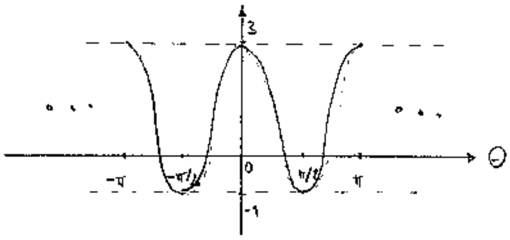
$$y[n] = x[n] + x[n-2] + x[n+2]$$

$\Rightarrow$ zeitinvariant

A 2.9

(b) $h[n] = \delta[n] + \delta[n+2] + \delta[n-2] \quad \circ\!-\!\bullet \quad 1 + e^{j2\Theta} + e^{-j2\Theta} = 1 + 2\cos(2\Theta) = H(e^{j\Theta})$





(c) $y[n] = (-1)^n + (-1)^{n-2} + (-1)^{n+2} = 3 \cdot (-1)^n$

Bemerkung: $(-1)^n = \frac{1}{2}\left(e^{j\pi n} + e^{-j\pi n}\right) = \cos(\pi n)$

(d) $y[n] = \lambda^n + \lambda^{n-2} + \lambda^{n+2} = \lambda^n \cdot \left(1 + \lambda^{-2} + \lambda^2\right) = 0$

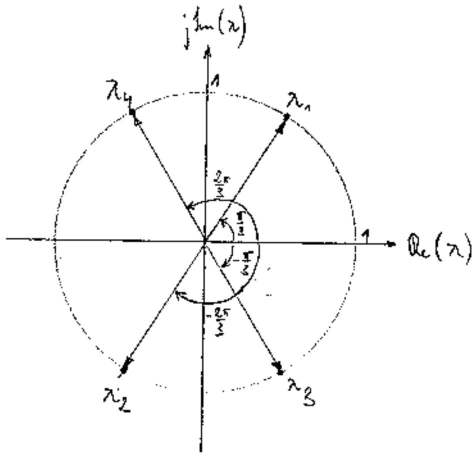
$(\lambda^n = 0)$

$1 + \lambda^{-2} + \lambda^2 = 0$

$\lambda^4 + \lambda^2 + 1 = 0$

$\lambda^2_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - 1} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2} = e^{\pm j\frac{2\pi}{3}}$

$\lambda_{1,2,3,4} = \pm\sqrt{-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}}$



(e) reales System: kausal ⇒ Anpassung notwendig

$y[n] = x[n] + x[n-2] + x[n-4]$