A 3.7

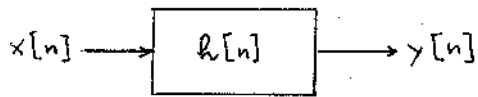$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$x[n] = \left(\frac{1}{2}\right)^n \sigma[n] - \frac{1}{4}\left(\frac{1}{2}\right)^{n-1} \sigma[n-1] = x_1[n] + a \cdot x_2[n] \quad ; \quad a = -\frac{1}{4} \quad ; \quad x_2[n] = x_1[n-1]$$

$$y[n] = \left(\frac{1}{3}\right)^n \sigma[n]$$

$$y[n] = (x * h)[n]$$

(a) $\quad H(e^{j\Theta}) = \dfrac{Y(e^{j\Theta})}{X(e^{j\Theta})}$

$$Y(e^{j\Theta}) = \frac{1}{1-\frac{1}{3}e^{-j\Theta}} \qquad\qquad X(e^{j\Theta}) = \frac{1}{1-\frac{1}{2}e^{-j\Theta}} - \frac{1}{4}e^{-j\Theta} \cdot \frac{1}{1-\frac{1}{2}e^{-j\Theta}} = \frac{1}{1-\frac{1}{2}e^{-j\Theta}} \cdot \left\{1 - \frac{1}{4}e^{-j\Theta}\right\}$$

$$H(e^{j\Theta}) = \frac{1}{1-\frac{1}{3}e^{-j\Theta}} \cdot \frac{1-\frac{1}{2}e^{-j\Theta}}{1-\frac{1}{4}e^{-j\Theta}} = \frac{A}{1-\frac{1}{3}e^{-j\Theta}} + \frac{B}{1-\frac{1}{4}e^{-j\Theta}}$$

$$A = -2 \qquad\qquad B = 3$$

$$H(e^{j\Theta}) = -2 \frac{1}{1-\frac{1}{3}e^{-j\Theta}} + 3 \frac{1}{1-\frac{1}{4}e^{-j\Theta}}$$

(b) $\quad h[n] \circ\!\!-\!\!\bullet H(e^{j\Theta})$

$$h[n] = -2\left(\frac{1}{3}\right)^n \sigma[n] + 3\left(\frac{1}{4}\right)^n \sigma[n] = \alpha\, h_1[n] + \beta\, h_2[n]$$

(c) $\quad \alpha = -2 \qquad\qquad \beta = 3$

$$y_2[n] = \frac{1}{4}\, y_2[n-1] + x_2[n-1]$$

$$h_1[n] \circ\!\!-\!\!\bullet H_1(e^{j\Theta}) = \frac{Y_1(e^{j\Theta})}{X_1(e^{j\Theta})}$$

$$Y_1(e^{j\Theta}) = X_1(e^{j\Theta}) \cdot H_1(e^{j\Theta})$$

$$\rightarrow Y_1(e^{j\Theta}) = \frac{1}{1-\frac{1}{3}e^{-j\Theta}} X(e^{j\Theta})$$

$$Y_1(e^{j\Theta}) = \frac{1}{3} Y_1(e^{j\Theta}) \cdot e^{-j\Theta} + X(e^{j\Theta})$$

$$y_1[n] = \frac{1}{3} y_1[n-1] + x[n]$$