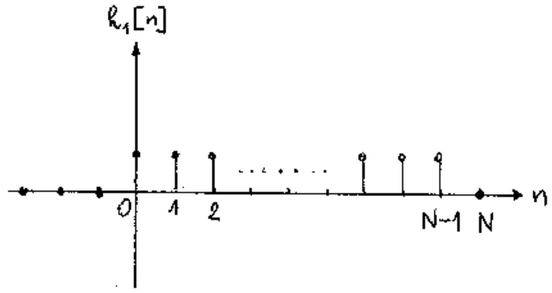
A 2.6

(a) (i) $$z[n] = \sum_{k=n-N+1}^{n} x[k] = \sum_{k=0}^{N-1} x[n-k]$$

$$h_1[n] = \sum_{k=0}^{N-1} \delta[n-k] =$$

$$= \sigma[n] - \sigma[n-N]$$



(ii) $$h[n] = (h_1 * h_2)[n] = \sum_{k=-\infty}^{\infty} 2 \cdot \sin\left(\frac{\pi}{N}\right) \cdot \sin\left(\frac{2\pi k}{N}\right) \cdot \sigma[k] \cdot \sum_{\ell=0}^{N-1} \delta[n-k-\ell] =$$

$$= \sum_{k=-\infty}^{\infty} 2 \sin\left(\frac{\pi}{N}\right) \cdot \sin\left(\frac{2\pi(n-k)}{N}\right) \sigma[n-k] \sum_{\ell=0}^{N-1} \delta[k-\ell] =$$

$$= \sum_{\ell=0}^{N-1} 2 \sin\left(\frac{\pi}{N}\right) \sin\left(\frac{2\pi(n-\ell)}{N}\right) \sigma[n-\ell]$$

$$h[n] = \sum_{k=-\infty}^{\infty} 2 \sin\left(\frac{\pi}{N}\right) \sin\left(\frac{2\pi k}{N}\right) \sigma[k] \cdot \left(\sigma[n-k] - \sigma[n-N-k]\right) =$$

$$= 2 \sin\left(\frac{\pi}{N}\right) \cdot \left\{ \sigma[n] \sum_{k=0}^{n} \sin\left(\frac{2\pi k}{N}\right) - \sigma[n-N] \sum_{k=0}^{n-N} \sin\left(\frac{2\pi k}{N}\right) \right\} =$$

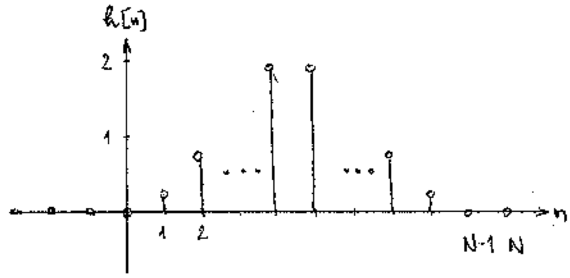
$$\sum_{k=0}^{n} \sin\left(\frac{2\pi k}{N}\right) = \frac{1}{2j} \sum_{k=0}^{n} e^{j\frac{2\pi k}{N}} - e^{-j\frac{2\pi k}{N}} = \frac{1}{2j} \left\{ \frac{1 - e^{j\frac{2\pi}{N}(n+1)}}{1 - e^{j\frac{2\pi}{N}}} - \frac{1 - e^{-j\frac{2\pi}{N}(n+1)}}{1 - e^{-j\frac{2\pi}{N}}} \right\} =$$

$$= \frac{1}{2j} \left\{ e^{j\frac{\pi}{N}n} \frac{\sin\left(\frac{\pi}{N}(n+1)\right)}{\sin\left(\frac{\pi}{N}\right)} - e^{-j\frac{\pi}{N}n} \frac{\sin\left(\frac{\pi}{N}(n+1)\right)}{\sin\left(\frac{\pi}{N}\right)} \right\} =$$

$$= \frac{1}{\sin\left(\frac{\pi}{N}\right)} \sin\left(\frac{\pi}{N} n\right) \sin\left(\frac{\pi}{N}(n+1)\right)$$

$$\sum_{k=0}^{n-N} \sin\left(\frac{2\pi k}{N}\right) = \frac{1}{\sin\left(\frac{\pi}{N}\right)} \sin\left(\frac{\pi}{N} n\right) \sin\left(\frac{\pi}{N}(n+1)\right)$$

$$h[n] = 2 \sin\left(\frac{\pi}{N} n\right) \sin\left\{\frac{\pi}{N}(n+1)\right\} \cdot \left(\sigma[n] - \sigma[n-N]\right)$$



A 2.6

(b) $x[n] = \sin\left(\frac{2\pi}{N}n\right)$

$$(h_1 * x)[n] = \sum_{k=-\infty}^{\infty} \sum_{\ell=0}^{N-1} \delta[k-\ell] \sin\left(\frac{2\pi}{N}(n-k)\right) = \sum_{\ell=0}^{N-1} \sin\left(\frac{2\pi}{N}(n-\ell)\right) =$$

$$= \sum_{\ell=0}^{N-1} \left\{ \sin\left(\frac{2\pi}{N}n\right) \cos\left(\frac{2\pi}{N}\ell\right) - \cos\left(\frac{2\pi}{N}n\right) \sin\left(\frac{2\pi}{N}\ell\right) \right\} = 0$$

$$(0 * h_2)[n] = 0$$

$$y[n] = 0$$

(c) $$(h_2 * x)[n] = \sum_{k=-\infty}^{\infty} 2 \sin\left(\frac{\pi}{N}\right) \sin\left(\frac{2\pi}{N}k\right) \sigma[k] \cdot \sin\left(\frac{2\pi}{N}(n-k)\right) =$$

$$= 2 \sin\left(\frac{\pi}{N}\right) \left\{ \sin\left(\frac{2\pi}{N}n\right) \sum_{k=0}^{\infty} \sin\left(\frac{2\pi}{N}k\right) \cos\left(\frac{2\pi}{N}k\right) - \cos\left(\frac{2\pi}{N}n\right) \sum_{k=0}^{\infty} \sin^2\left(\frac{2\pi}{N}k\right) \right\} = \text{nicht konvergent!}$$

Vertauschung d. Teilsysteme wegen Teilinstabilität nicht zulässig