A 3.9

(a) $Y(e^{j\Theta}) = X(e^{j\Theta}) \cdot H(e^{j\Theta}) = X(e^{j\Theta}) + \alpha e^{-j\Theta} X(e^{j\Theta}) + \beta \cdot e^{-j2\Theta} X(e^{j\Theta})$

$$e^{-j\Theta N_0} X(e^{j\Theta}) \bullet\!\!-\!\!\circ \; x[n-N_0]$$

$$y[n] = x[n] + \alpha x[n-1] + \beta x[n-2]$$

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$$y[n] = \cos(\Theta_0 n) + \alpha \cos(\Theta_0 n - \Theta_0) + \beta \cos(\Theta_0 n - 2\Theta_0) = 0$$

$$\cos(\eta - \xi) = \cos(\eta)\cos(\xi) + \sin(\eta)\sin(\xi)$$

$$y[n] = \cos(\Theta_0 n) + \alpha \cos(\Theta_0 n)\cos(\Theta_0) + \alpha \sin(\Theta_0 n)\sin(\Theta_0) + \beta \cos(\Theta_0 n)\cos(2\Theta_0) + \beta \sin(\Theta_0 n)\sin(2\Theta_0) =$$

$$= \cos(\Theta_0 n) \cdot \left[\alpha \cos(\Theta_0) + \beta \cos(2\Theta_0) + 1\right] + \sin(\Theta_0 n) \cdot \left[\alpha \sin(\Theta_0) + \beta \sin(2\Theta_0)\right]$$

$$y[n] = 0 \quad \forall n$$

I: $1 + \alpha \cos(\Theta_0) + \beta \cos(2\Theta_0) = 0$

II: $\alpha \sin(\Theta_0) + \beta \sin(2\Theta_0) = 0$ $\qquad \alpha = -\beta \dfrac{\sin(2\Theta_0)}{\sin(\Theta_0)}$

$$1 - \beta \sin(2\Theta_0) \frac{\cos(\Theta_0)}{\sin(\Theta_0)} + \beta \cos(2\Theta_0) = 0$$

$$\sin(2\Theta_0) = 2\sin(\Theta_0)\cos(\Theta_0)$$

$$\cos(2\Theta_0) = 2\cos^2(\Theta_0) - 1$$

$$1 \cancel{-2\beta\cos^2(\Theta_0)} + \cancel{2\beta\cos^2(\Theta_0)} - \beta = 0$$

$$\beta = 1$$

$$\alpha = -2\cos(\Theta_0)$$

A 3.9

(b) $H(e^{j\theta}) = 1 - 2\cos(\theta_0)\, e^{-j\theta} + e^{-j2\theta} = e^{-j\theta} \cdot \left\{ e^{j\theta} + e^{-j\theta} - 2\cos(\theta_0) \right\} = 2 e^{-j\theta} \left\{ \cos(\theta) - \cos(\theta_0) \right\}$

$|H(e^{j\theta})| = 2 \, | \cos(\theta) - \cos(\theta_0) |$

$\mathrm{arc}\left\{ H(e^{j\theta}) \right\} = -\theta + \mathrm{arc}\left\{ \cos(\theta) - \cos(\theta_0) \right\}$

2. b). $\theta_0 = \frac{\pi}{3}$