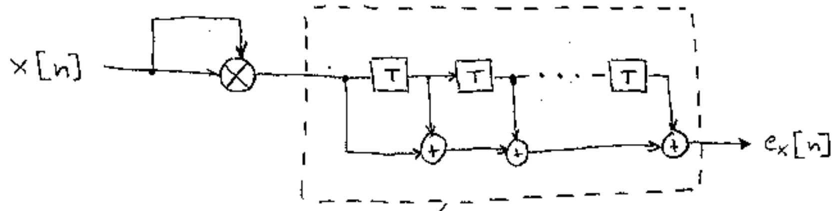
A 3.10

$$e_x[n] = \sum_{k=n-N+1}^{n} x^2[k] = \sum_{k=0}^{N-1} x^2[n-k]$$

(a)




$h[n] \circ\!\!-\!\!\bullet\, H(e^{j\Theta})$

(b) $e_x[n] = x^2[n] * h[n] \circ\!\!-\!\!\bullet\, E_x(e^{j\Theta}) = \frac{1}{2\pi} \cdot (X * X)(e^{j\Theta}) \cdot H(e^{j\Theta})$

$$h[n] = \sum_{k=0}^{N-1} x_1[n-k] \circ\!\!-\!\!\bullet \sum_{k=0}^{N-1} e^{-j\Theta k} X_1(e^{j\Theta}) = X_1(e^{j\Theta}) \cdot \sum_{k=0}^{N-1} e^{-j\Theta k} =$$

$$= X_1(e^{j\Theta}) \frac{1-e^{-j\Theta N}}{1-e^{-j\Theta}} = X_1(e^{j\Theta}) \frac{e^{-j\Theta\frac{N}{2}}}{e^{-j\frac{\Theta}{2}}} \frac{e^{j\Theta\frac{N}{2}} + e^{-j\Theta\frac{N}{2}}}{e^{j\frac{\Theta}{2}} - e^{-j\frac{\Theta}{2}}} =$$

$$= X_1(e^{j\Theta}) \cdot \frac{\sin\left(\frac{\Theta N}{2}\right)}{\sin\left(\frac{\Theta}{2}\right)} \cdot e^{-j\frac{\Theta}{2}(N-1)}$$

$$E_x(e^{j\Theta}) = \frac{1}{2\pi} (X * X)(e^{j\Theta}) \cdot \frac{\sin\left(\frac{\Theta N}{2}\right)}{\sin\left(\frac{\Theta}{2}\right)} e^{-j\frac{\Theta}{2}(N-1)}$$

A 3.10

(c) $x[n] = A\cos(\Theta_0 n)$

$e_x[n] = \text{const.}$

$$e_x[n] = \sum_{k=0}^{N-1} A^2 \cos^2(\Theta_0(n-k)) = A^2 \frac{1}{2} \sum_{k=0}^{N-1} 1 + \cos(2\Theta_0(n-k)) = \frac{A^2}{2} \cdot \left\{ N + \sum_{k=0}^{N-1} \cos(2\Theta_0(n-k)) \right\} = \text{const.}$$

$$\sum_{k=0}^{N-1} \cos(2\Theta_0(n-k)) = \frac{1}{2} \sum_{k=0}^{N-1} e^{j2\Theta_0 n} e^{-j2\Theta_0 k} + e^{-j2\Theta_0 n} e^{j2\Theta_0 k} =$$

$$= \frac{1}{2} \cdot \left\{ e^{j2\Theta_0 n} \frac{1-e^{-j2\Theta_0 N}}{1-e^{-j2\Theta_0}} + e^{-j2\Theta_0 n} \frac{1-e^{j2\Theta_0 N}}{1-e^{j2\Theta_0}} \right\} = \text{const.}$$

Forderung: $\left.\begin{array}{l} 1-e^{-j2\Theta_0 N} = 0 \\ 1-e^{j2\Theta_0 N} = 0 \end{array}\right\} \quad 2\Theta_0 N = 2\pi \cdot \ell \, , \ \ell \in \mathbb{Z} \Rightarrow N = \frac{\pi}{\Theta_0} \cdot \ell \, , \ N \in \mathbf{N}$