A 1.7

(a) $x[n] = 1 - \cos[\frac{\pi}{4}n] = e^{j\frac{2\pi\cdot 0}{8}n} - \cos[\frac{2\pi}{8}n]$

$c_k = \delta_8[k] - \left(\frac{1}{2}\delta_8[k-m] + \frac{1}{2}\delta_8[k+m]\right)$ mit $m=1$

$c_k = \delta_8[k] - \frac{1}{2}\delta_8[k-1] - \frac{1}{2}\delta_8[k+1]$

(b) $c_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi k}{N}n} = \frac{1}{6}\sum_{k=-2}^{3}\left(\frac{1}{2}\right)^n e^{-j\frac{2\pi k}{6}n} =$

$= \frac{1}{6}\sum_{n=-2}^{3}\left(\frac{1}{2}e^{-j\frac{2\pi k}{6}}\right)^n = \frac{1}{6}\sum_{n=0}^{5}\left(\frac{1}{2}e^{-j\frac{\pi k}{3}}\right)^{n-2} =$

$= \frac{2}{3}e^{j\frac{2\pi k}{3}}\sum_{n=0}^{5}\left(\frac{1}{2}e^{-j\frac{\pi k}{3}}\right)^n = \frac{2}{3}e^{j\frac{2\pi k}{3}}\frac{1-\left(\frac{1}{2}\right)^6}{1-\frac{1}{2}e^{-j\frac{\pi k}{3}}}$

(c) $x[n] = \sum_{k=-\infty}^{\infty}\delta[n-3k]$

$c_k = \frac{1}{3}$

(d) $x[n] = e^{j\frac{2\pi L}{M}n} = e^{j\frac{2\pi}{N}\cdot\frac{L}{M}\cdot N\cdot n}$

$c_k = \delta_N[k - \frac{L}{M}N]$

(e) $x[n] = \sum_{k=-\infty}^{\infty}\delta[n-8k] + \sum_{k=-\infty}^{\infty}\delta[n-8k-1] + \sum_{k=-\infty}^{\infty}\delta[n-8k-2] + \sum_{k=-\infty}^{\infty}\delta[n-8k+1] + \sum_{k=-\infty}^{\infty}\delta[n-8k+2] =$

$= \sum_{\ell=-2}^{2}\sum_{k=-\infty}^{\infty}\delta[n-8k-\ell]$

1.7

(e) weiter

$$x[n] = \sum_{\ell=-2}^{2} \sum_{k=-\infty}^{\infty} \delta[n-8k-\ell]$$

$$c_k = \sum_{\ell=-2}^{2} e^{-j\frac{2\pi k}{8}\cdot \ell} \cdot \frac{1}{8} = \frac{1}{8} \sum_{\ell=0}^{4} e^{-j\frac{\pi k}{4}(\ell-2)} =$$

$$= \frac{1}{8} e^{j\frac{\pi}{2}k} \sum_{\ell=0}^{4} e^{-j\frac{\pi}{4}k\ell} = \frac{1}{8} e^{j\frac{\pi}{2}k} \cdot \frac{1-e^{-j\frac{5\pi}{4}k}}{1-e^{-j\frac{k\pi}{4}}} =$$

$$= \frac{1}{8} e^{j\frac{\pi}{2}k} \frac{e^{-j\frac{5\pi}{8}k}}{e^{-j\frac{\pi}{8}k}} \frac{e^{j\frac{5\pi}{8}k} - e^{-j\frac{5\pi}{8}k}}{e^{j\frac{\pi}{8}k} - e^{-j\frac{\pi}{8}k}} = \frac{1}{8} \frac{\sin\left[\frac{5\pi}{8}k\right]}{\sin\left[\frac{\pi}{8}k\right]}$$