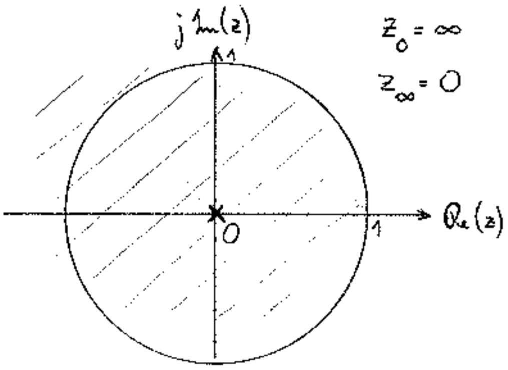
A4.1

(a) $\delta[n-1] \circ\!\!-\!\!\bullet\ z^{-1} = \frac{1}{z}$

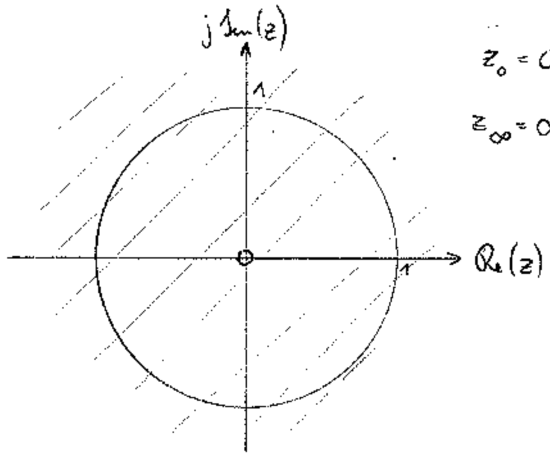


$z_0 = \infty$

$z_\infty = 0$

$0 < R \leq \infty$

(b) $\delta[n+1] \circ\!\!-\!\!\bullet\ z$

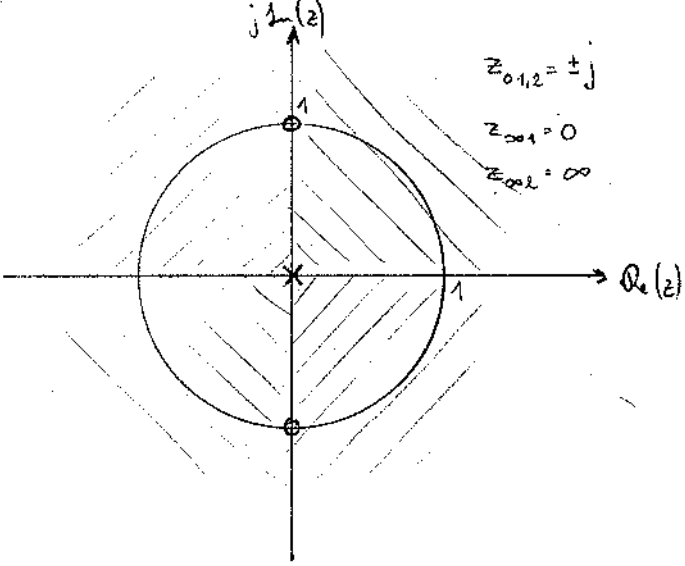


$z_0 = 0$

$z_\infty = \infty$

$0 \leq R < \infty$

(c) $\delta[n-1] + \delta[n+1] \circ\!\!-\!\!\bullet\ \frac{1}{z} + z = \frac{z^2+1}{z}$



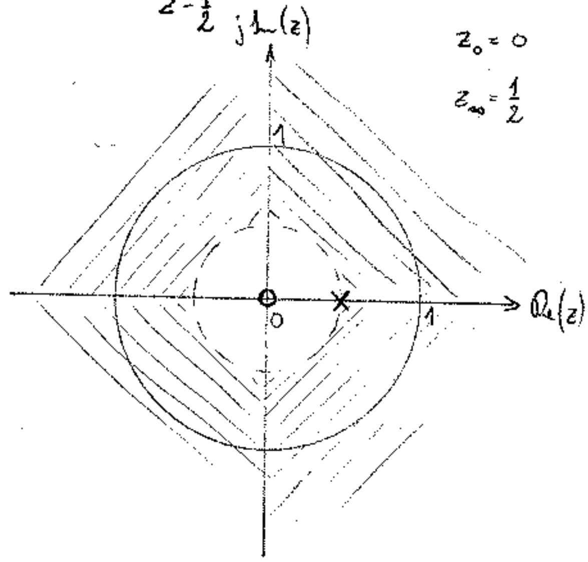
$z_{0\,1,2} = \pm j$

$z_{\infty 1} = 0$

$z_{\infty 2} = \infty$

$0 < R < \infty$

(d) $\left(\frac{1}{2}\right)^n \sigma[n] \circ\!\!-\!\!\bullet\ \frac{z}{z-\frac{1}{2}}$

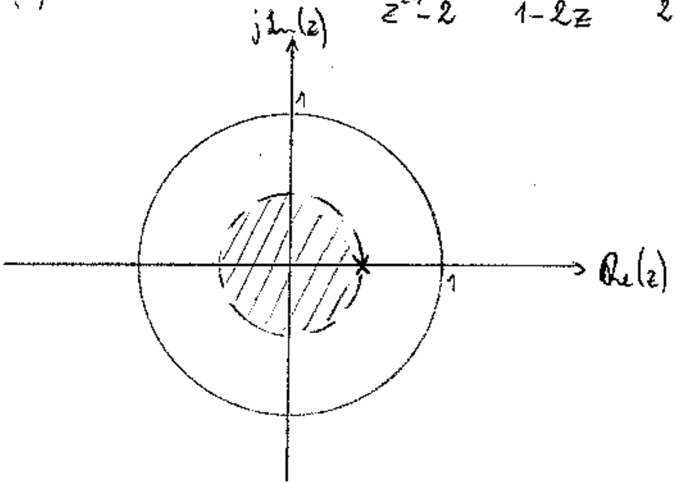


$z_0 = 0$

$z_\infty = \frac{1}{2}$

$\frac{1}{2} < R \leq \infty$

(e) $\left(\frac{1}{2}\right)^n \sigma[-n] = 2^{-n} \sigma[-n] \circ\!\!-\!\!\bullet\ \frac{z^{-1}}{z^{-1}-2} = \frac{1}{1-2z} = -\frac{1}{2}\,\frac{1}{z-\frac{1}{2}}$



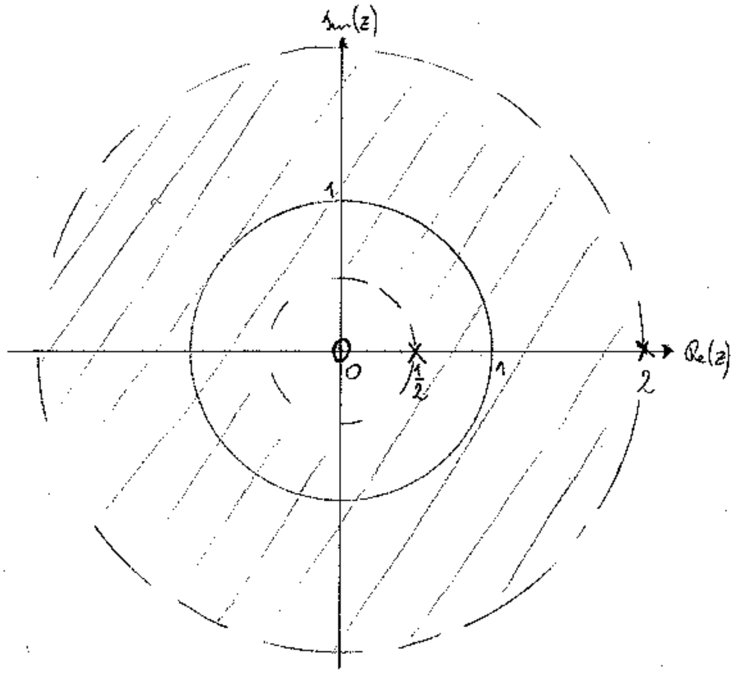
$z_0 = \infty$

$z_\infty = \frac{1}{2}$

$0 \leq R < \frac{1}{2}$

A 4.1

(f) $\left(\frac{1}{2}\right)^{|n|} = \left(\frac{1}{2}\right)^{-n}\sigma[-n] - \delta[n] + \left(\frac{1}{2}\right)^{n}\sigma[n] \quad \circ\!\!-\!\!\bullet \quad \frac{z^{-1}}{z^{-1}-\frac{1}{2}} - 1 + \frac{z}{z-\frac{1}{2}} = -\frac{3}{2}\,\frac{z}{(z-2)(z-\frac{1}{2})}$



$z_{\infty 1} = \frac{1}{2} \qquad z_{\infty 2} = 2$

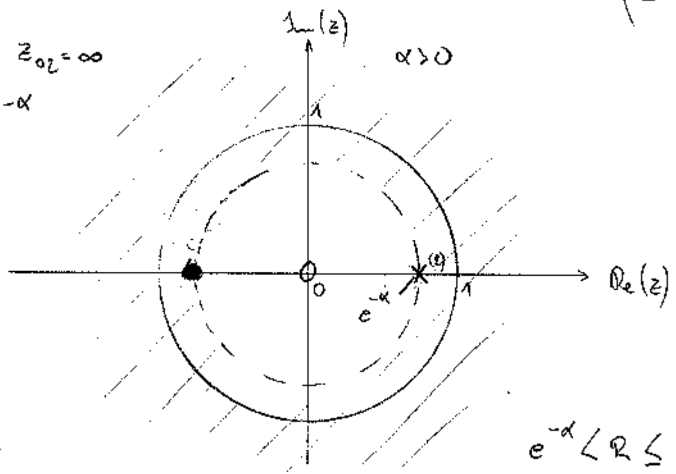
$z_{01} = 0 \qquad z_{02} = \infty$

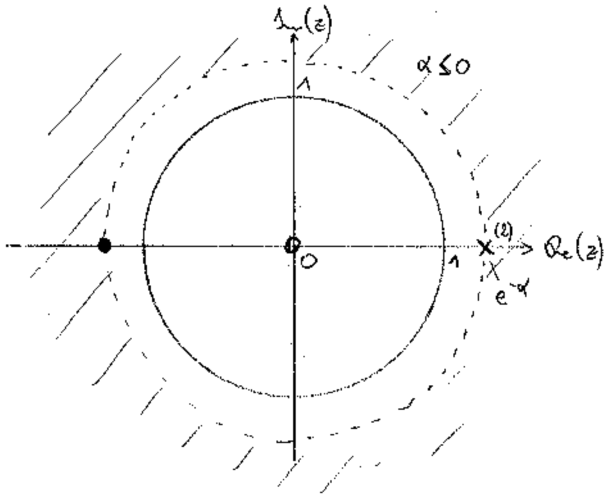
$\frac{1}{2} < R < 2$

(g) $n\, e^{-\alpha n}\sigma[n] = n\cdot\left(e^{-\alpha}\right)^{n}\sigma[n] \quad \circ\!\!-\!\!\bullet \quad -z\cdot\frac{d}{dz}\left(\frac{z}{z-e^{-\alpha}}\right) = -z\,\frac{z-e^{-\alpha}-z}{\left(z-e^{-\alpha}\right)^2} = +\frac{z\cdot e^{-\alpha}}{\left(z-e^{-\alpha}\right)^2}$

$z_{01}=0 \qquad z_{02}=\infty$

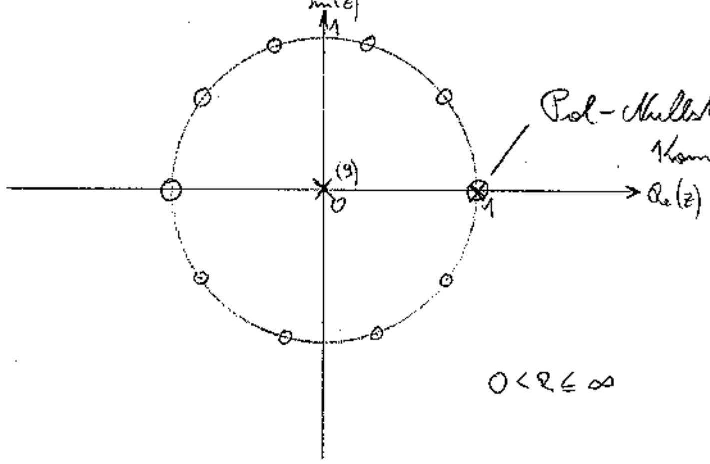
$z_{\infty 1,2}=e^{-\alpha}$





$e^{-\alpha} < R \leq \infty$

(h) $x[n] = \sigma[n] - \sigma[n-10] \quad \circ\!\!-\!\!\bullet \quad \frac{z}{z-1} - z^{-10}\,\frac{z}{z-1} = \frac{z-z^{-9}}{z-1} = \frac{z^{10}-1}{z^{9}(z-1)}$



Pol-Nullstellen-Kompensation!

$z^{10} = 1$

$\left(r e^{j\Theta}\right)^{10} = 1$

$\Rightarrow r = 1$

$10\,\Theta = 2\pi\cdot l$

$\Theta = \frac{2\pi}{10}\cdot l \,, \quad l = 0,\dots,9$

$0 < R \leq \infty$

$z_{\infty 1,\dots,9} = 0$

$z_{0 1,\dots,9} = e^{j\frac{2\pi}{10}\cdot l} \,, \quad l = 1,\dots,9$