Biometrie-Aufgabe bei Prof. Hasibeder SS05
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Die Fehlerfreiheit dieser Lösungen kann nicht garantiert werden!

Logrank Test

$$\chi^2 = \frac{(d_1 - e_1)^2}{e_1} + \frac{(d_2 - e_2)^2}{e_2}$$

mit

$$e_1 = \sum_{i=1}^{k} (d_{1i} + d_{2i}) \cdot \frac{n_{1i}}{n_{1i} + n_{2i}} \text{ und } e_2 = \sum_{i=1}^{k} (d_{1i} + d_{2i}) \cdot \frac{n_{2i}}{n_{1i} + n_{2i}}$$

Aufgabe 20 (25.5.2005)

Führen Sie einen Logrank Test für die folgenden beiden Stichproben durch:

Stichprobe 1 (n = 12):
Endereignis: 7, 15, 27, 48 (x2), 63
Zensur: 30, 50, 56

Stichprobe 2 (n = 38):
Endereignis: 15, 36, 42, 43, 44, 48, 50 (x3), 70, 75
Zensur: 10 (x3), 20 (x5), 40, 43 (x5)

(Notwendige Spalten: Endereignis, Zensierte Daten, ni, di, ni-di. Dann in Formel einsetzen.)
H0: S1(x) = S2(x)

Freiheitsgrade: f= 2-1 = 1
-> Prüfgröße bei α = 10% ist $\chi^2 = 2{,}706$

Stichprobe 1:

Endereignis	ci	ni	di	ni-di
t = 7	0	12	1	11
t = 15	0	11	1	10
t =27	0	10	1	9
t = 48 (*2)	1	8	2	6
t = 63	2	4	1	3

Stichprobe 2:

Endereignis	ci	ni	di	ni-di
t =15	3	35	1	34
t = 36	5	29	1	28
t = 42	1	27	1	26
t = 43	0	26	1	25
t = 44	5	20	1	19
t = 48	0	19	1	18
t = 50	0	18	3	15
t = 70	0	15	1	14
t = 75	0	14	1	13

Beide Stichproben:

Endereignis	ci	ni	di	ni-di
t = 7	0	n1i = 12 n2i = 38	d1i = 1	(n1i-d1i) = 11 (n2i-d2i) = 38
t =15	c2i = 3	n1i = 11 n2i = 35	d1i = 1 d2i = 1	(n1i-d1i) = 10 (n2i-d2i) = 34
t = 27	c2i = 5	n1i = 10 n2i = 29	d1i = 1	(n1i-d1i) = 9 (n2i-d2i) = 29
t = 36	c1i = 1	n1i = 8 n2i = 29	d2i = 1	(n1i-d1i) = 8 (n2i-d2i) = 28
t = 42	c2i = 1	n1i = 8 n2i = 27	d2i = 1	(n1i-d1i) = 8 (n2i-d2i) = 26
t = 43	0	n1i = 8 n2i = 26	d2i = 1	(n1i-d1i) = 8 (n2i-d2i) = 25
t = 44	c2i = 5	n1i = 8 n2i = 20	d2i = 1	(n1i-d1i) = 8 (n2i-d2i) = 19
t = 48	0	n1i = 8 n2i = 19	d1i = 2 d2i = 1	(n1i-d1i) = 6 (n2i-d2i) = 18
t = 50	0	n1i = 6 n2i = 18	d2i = 3	(n1i-d1i) = 6 (n2i-d2i) = 15
t = 63	c1i = 2	n1i = 4 n2i = 15	d1i = 1	(n1i-d1i) = 3 (n2i-d2i) = 15
t = 70	0	n1i = 3 n2i = 15	d2i = 1	(n1i-d1i) = 3 (n2i-d2i) = 14
t = 75	0	n1i = 3 n2i = 14	d2i = 1	(n1i-d1i) = 3 (n2i-d2i) = 13

$$e_1 = \sum_{i=1}^{k}(d_{1i}+d_{2i})\cdot\frac{n_{1i}}{n_{1i}+n_{2i}} = \left(1\cdot\frac{12}{12+38}\right)+\left(2\cdot\frac{11}{11+35}\right)+\left(1\cdot\frac{10}{10+29}\right)+\left(1\cdot\frac{8}{8+29}\right)+\left(1\cdot\frac{8}{8+27}\right)+$$
$$\left(1\cdot\frac{8}{8+26}\right)+\left(1\cdot\frac{8}{8+20}\right)+\left(3\cdot\frac{8}{8+19}\right)+\left(3\cdot\frac{6}{6+18}\right)+\left(1\cdot\frac{4}{4+15}\right)+\left(1\cdot\frac{3}{3+15}\right)+\left(1\cdot\frac{3}{3+14}\right)\approx 4{,}133$$

$$e_2 = \sum_{i=1}^{k}(d_{1i}+d_{2i})\cdot\frac{n_{2i}}{n_{1i}+n_{2i}} = \left(1\cdot\frac{38}{12+38}\right)+\left(2\cdot\frac{35}{11+35}\right)+\left(1\cdot\frac{29}{10+29}\right)+\left(1\cdot\frac{29}{8+29}\right)+\left(1\cdot\frac{27}{8+27}\right)+$$
$$\left(1\cdot\frac{26}{8+26}\right)+\left(1\cdot\frac{20}{8+20}\right)+\left(3\cdot\frac{19}{8+19}\right)+\left(3\cdot\frac{18}{6+18}\right)+\left(1\cdot\frac{15}{4+15}\right)+\left(1\cdot\frac{15}{3+15}\right)+\left(1\cdot\frac{14}{3+14}\right)\approx 12{,}867$$

$$\chi^2 = \frac{(d_1-e_1)^2}{e_1}+\frac{(d_2-e_2)^2}{e_2} = \frac{(6-4{,}133)^2}{4{,}133}+\frac{(11-12{,}867)^2}{12{,}867}\approx 0{,}9528$$

Für α = 10% ist $\chi^2 < 2{,}706$

Die Nullhypothese lässt sich also nicht verwerfen (auch nicht für kleineres α). Die beiden Stichproben haben also in etwa dieselbe Sterbeverteilung.