Anhang C: Konfidenzintervalle

1. $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$, ua.

Parameter	Voraussetzung		$100(1-\alpha)\%$ Intervall
μ	σ bekannt	zweiseitig	$\overline{X} \pm u_{1-\alpha/2} \frac{\sigma}{\sqrt{n}}$
		unteres	$\left(-\infty, \overline{X} + u_{1-\alpha} \frac{\sigma}{\sqrt{n}}\right]$
		oberes	$\left[\overline{X} - u_{1-\alpha} \frac{\sigma}{\sqrt{n}}, \infty\right)$
	σ unbekannt	zweiseitig	$\overline{X} \pm t_{n-1;\,1-\alpha/2} \frac{S}{\sqrt{n}}$
		unteres	$\left(-\infty, \overline{X} + t_{n-1;\,1-\alpha} \frac{S}{\sqrt{n}}\right]$
		oberes	$\left[\overline{X} - t_{n-1;\,1-\alpha} \frac{S}{\sqrt{n}}, \infty\right)$
σ^2	μ unbekannt	zweiseitig	$\left[\frac{(n-1)S^2}{\chi^2_{n-1;\,1-\alpha/2}}, \frac{(n-1)S^2}{\chi^2_{n-1;\,\alpha/2}}\right]$
		unteres	$\left(0, \frac{(n-1)S^2}{\chi^2_{n-1;\,\alpha}}\right]$
		oberes	$\left[\frac{(n-1)S^2}{\chi^2_{n-1;\,1-\alpha}}, \infty\right)$
σ	μ unbekannt	zweiseitig	$\left[\frac{\sqrt{n-1}\,S}{\sqrt{\chi^2_{n-1;\,1-\alpha/2}}}, \frac{\sqrt{n-1}\,S}{\sqrt{\chi^2_{n-1;\,\alpha/2}}}\right]$
		unteres	$\left(0, \frac{\sqrt{n-1}\,S}{\sqrt{\chi^2_{n-1;\,\alpha}}}\right]$
		oberes	$\left[\frac{\sqrt{n-1}\,S}{\sqrt{\chi^2_{n-1;\,1-\alpha}}}, \infty\right)$

2. $X_1, \ldots, X_n \sim N(\mu_1, \sigma_1^2)$, $Y_1, \ldots, Y_m \sim N(\mu_2, \sigma_2^2)$, ua.

Parameter	Voraussetzung		$100(1-\alpha)\%$ Intervall
$\mu_1 - \mu_2$	σ_1, σ_2 bekannt	zweiseitig	$\overline{X} - \overline{Y} \pm u_{1-\alpha/2} \sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}$
		unteres	$\left(-\infty, \overline{X} - \overline{Y} + u_{1-\alpha} \sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}\right]$
		oberes	$\left[\overline{X} - \overline{Y} - u_{1-\alpha} \sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}, \infty\right)$
	σ_1, σ_2 unbekannt $\sigma_1 = \sigma_2$	zweiseitig	$\overline{X} - \overline{Y} \pm t_{n+m-2;\,1-\alpha/2}\, S_p \sqrt{\frac{1}{n} + \frac{1}{m}}$ mit $S_p^2 = \frac{(n-1)S_1^2 + (m-1)S_2^2}{n+m-2}$
		unteres	$\left(-\infty, \overline{X} - \overline{Y} + t_{n+m-2;\,1-\alpha}\, S_p \sqrt{\frac{1}{n} + \frac{1}{m}}\right]$
		oberes	$\left[\overline{X} - \overline{Y} - t_{n+m-2;\,1-\alpha}\, S_p \sqrt{\frac{1}{n} + \frac{1}{m}}, \infty\right)$

3. $X_1, \ldots, X_n \sim Ex_\tau$, ua.

Parameter		$100(1-\alpha)\%$ Intervall
τ	zweiseitig	$\left[\frac{2n\overline{X}}{\chi^2_{2n;\,1-\alpha/2}}, \frac{2n\overline{X}}{\chi^2_{2n;\,\alpha/2}}\right]$
	unteres	$\left(0, \frac{2n\overline{X}}{\chi^2_{2n;\,\alpha}}\right]$
	oberes	$\left[\frac{2n\overline{X}}{\chi^2_{2n;\,1-\alpha}}, \infty\right)$

4. $X_1, \ldots, X_n \sim A_p$, ua.

Parameter		(approx.) $100(1-\alpha)\%$ Intervall (n 'groß')
p	zweiseitig	$\widehat{p} \pm u_{1-\alpha/2}\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$ mit $\widehat{p} = \overline{X}$
	unteres	$\left(0,\ \widehat{p} + u_{1-\alpha}\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}\right]$
	oberes	$\left[\widehat{p} - u_{1-\alpha}\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}},\ 1\right)$

5. $X_1, \ldots, X_n \sim A_{p_1}$, $Y_1, \ldots, Y_m \sim A_{p_2}$ ua.

Parameter		(approx.) $100(1-\alpha)\%$ Intervall (n, m 'groß')
$p_1 - p_2$	zweiseitig	$\widehat{p}_1 - \widehat{p}_2 \pm u_{1-\alpha/2}\sqrt{\frac{\widehat{p}_1(1-\widehat{p}_1)}{n} + \frac{\widehat{p}_2(1-\widehat{p}_2)}{m}}$ mit $\widehat{p}_1 = \overline{X}$, $\widehat{p}_2 = \overline{Y}$
	unteres	$\left(-1,\ \widehat{p}_1 - \widehat{p}_2 + u_{1-\alpha}\sqrt{\frac{\widehat{p}_1(1-\widehat{p}_1)}{n} + \frac{\widehat{p}_2(1-\widehat{p}_2)}{m}}\right]$
	oberes	$\left[\widehat{p}_1 - \widehat{p}_2 - u_{1-\alpha}\sqrt{\frac{\widehat{p}_1(1-\widehat{p}_1)}{n} + \frac{\widehat{p}_2(1-\widehat{p}_2)}{m}},\ 1\right)$

Anhang D: Parametertests (mit Fehlerw. 1.Art α)

1. $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$, ua.

H_0	H_1	H_0 verwerfen, falls:
	σ bekannt	
$\mu = \mu_0$	$\mu \neq \mu_0$	$\frac{\lvert\overline{X} - \mu_0\rvert}{\sigma/\sqrt{n}} > z_{1-\alpha/2}$
$\mu \leq \mu_0$	$\mu > \mu_0$	$\frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} > z_{1-\alpha}$
$\mu \geq \mu_0$	$\mu < \mu_0$	$\frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} < -z_{1-\alpha}$
	σ unbekannt	
$\mu = \mu_0$	$\mu \neq \mu_0$	$\frac{\lvert\overline{X} - \mu_0\rvert}{S/\sqrt{n}} > t_{n-1;\,1-\alpha/2}$
$\mu \leq \mu_0$	$\mu > \mu_0$	$\frac{\overline{X} - \mu_0}{S/\sqrt{n}} > t_{n-1;\,1-\alpha}$
$\mu \geq \mu_0$	$\mu < \mu_0$	$\frac{\overline{X} - \mu_0}{S/\sqrt{n}} < -t_{n-1;\,1-\alpha}$
	μ unbekannt	
$\sigma = \sigma_0$	$\sigma \neq \sigma_0$	$\frac{(n-1)S^2}{\sigma_0^2} \notin \left[\chi^2_{n-1;\,\alpha/2}, \chi^2_{n-1;\,1-\alpha/2}\right]$
$\sigma \leq \sigma_0$	$\sigma > \sigma_0$	$\frac{(n-1)S^2}{\sigma_0^2} > \chi^2_{n-1;\,1-\alpha}$
$\sigma \geq \sigma_0$	$\sigma < \sigma_0$	$\frac{(n-1)S^2}{\sigma_0^2} < \chi^2_{n-1;\,\alpha}$

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \qquad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} X_i^2 - n\overline{X}^2\right] = \sigma^2$$

↗ Stichprobenmittel

↑ Stichprobenvarianz

Streuung = σ

2. $X_1, \ldots, X_n \sim N(\mu_1, \sigma_1^2)$, $Y_1, \ldots, Y_m \sim N(\mu_2, \sigma_2^2)$, ua.

H_0	H_1	H_0 verwerfen, falls:
σ_1, σ_2 bekannt		
$\mu_1 = \mu_2$	$\mu_1 \neq \mu_2$	$\dfrac{\lvert\overline{X} - \overline{Y}\rvert}{\sqrt{\sigma_1^2/n + \sigma_2^2/m}} > z_{1-\alpha/2}$
$\mu_1 \leq \mu_2$	$\mu_1 > \mu_2$	$\dfrac{\overline{X} - \overline{Y}}{\sqrt{\sigma_1^2/n + \sigma_2^2/m}} > z_{1-\alpha}$
$\mu_1 \geq \mu_2$	$\mu_1 < \mu_2$	$\dfrac{\overline{X} - \overline{Y}}{\sqrt{\sigma_1^2/n + \sigma_2^2/m}} < -z_{1-\alpha}$
σ_1, σ_2 unbekannt: $\sigma_1 = \sigma_2$		
$\mu_1 = \mu_2$	$\mu_1 \neq \mu_2$	$\dfrac{\lvert\overline{X} - \overline{Y}\rvert}{S_p\sqrt{1/n + 1/m}} > t_{n+m-2;\,1-\alpha/2}$
		mit $S_p^2 = \dfrac{(n-1)S_1^2 + (m-1)S_2^2}{n+m-2}$
$\mu_1 \leq \mu_2$	$\mu_1 > \mu_2$	$\dfrac{\overline{X} - \overline{Y}}{S_p\sqrt{1/n + 1/m}} > t_{n+m-2;\,1-\alpha}$
$\mu_1 \geq \mu_2$	$\mu_1 < \mu_2$	$\dfrac{\overline{X} - \overline{Y}}{S_p\sqrt{1/n + 1/m}} < -t_{n+m-2;\,1-\alpha}$
μ_1, μ_2 unbekannt		
$\sigma_1 = \sigma_2$	$\sigma_1 \neq \sigma_2$	$\dfrac{S_1^2}{S_2^2} \notin \left[\dfrac{1}{F_{m-1,n-1;\,1-\alpha/2}},\, F_{n-1,m-1;\,1-\alpha/2}\right]$
$\sigma_1 \leq \sigma_2$	$\sigma_1 > \sigma_2$	$\dfrac{S_1^2}{S_2^2} > F_{n-1,m-1;\,1-\alpha}$
$\sigma_1 \geq \sigma_2$	$\sigma_1 < \sigma_2$	$\dfrac{S_1^2}{S_2^2} < \dfrac{1}{F_{m-1,n-1;\,1-\alpha}}$

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \qquad \overline{Y} = \frac{1}{m}\sum_{j=1}^{m} Y_j$$

$$S_1^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2, \qquad S_2^2 = \frac{1}{m-1}\sum_{j=1}^{m}\left(Y_j - \overline{Y}\right)^2$$

3. $X_1, \ldots, X_n \sim A_p$, ua.

H_0	H_1	H_0 verwerfen, falls:
	n 'groß'	
$p = p_0$	$p \neq p_0$	$\dfrac{\lvert\widehat{p} - p_0\rvert}{\sqrt{p_0(1-p_0)/n}} > z_{1-\alpha/2}$ mit $\widehat{p} = \overline{X}$
$p \leq p_0$	$p > p_0$	$\dfrac{\widehat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} > z_{1-\alpha}$
$p \geq p_0$	$p < p_0$	$\dfrac{\widehat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} < -z_{1-\alpha}$

4. $X_1, \ldots, X_n \sim A_{p_1}, Y_1, \ldots, Y_m \sim A_{p_2}$ ua.

H_0	H_1	H_0 verwerfen, falls:
	n, m 'groß'	
$p_1 = p_2$	$p_1 \neq p_2$	$\dfrac{\lvert\widehat{p}_1 - \widehat{p}_2\rvert}{\sqrt{\widehat{p}(1-\widehat{p})\left(1/n + 1/m\right)}} > z_{1-\alpha/2}$ mit $\widehat{p}_1 = \overline{X},\ \widehat{p}_2 = \overline{Y},\ \widehat{p} = \dfrac{n\,\widehat{p}_1 + m\,\widehat{p}_2}{n+m}$
$p_1 \leq p_2$	$p_1 > p_2$	$\dfrac{\widehat{p}_1 - \widehat{p}_2}{\sqrt{\widehat{p}(1-\widehat{p})\left(1/n + 1/m\right)}} > z_{1-\alpha}$
$p_1 \geq p_2$	$p_1 < p_2$	$\dfrac{\widehat{p}_1 - \widehat{p}_2}{\sqrt{\widehat{p}(1-\widehat{p})\left(1/n + 1/m\right)}} < -z_{1-\alpha}$