Matrix × Vektor (= Vektor)

$$A \cdot \underline{x} = \begin{pmatrix} \underline{a}_1 & \underline{a}_2 & \dots & \underline{a}_n \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \quad \begin{array}{l} A \in K^{m \times n} \\ \in K^n \\ \in K^{n \times 1} \end{array}$$

$\underline{a}_1 \dots \underline{a}_n$ Spalten von A

$$x_1 \cdot \underline{a}_1 + x_2 \cdot \underline{a}_2 + \dots + x_n \cdot \underline{a}_n \in K^m$$

= Linearkombi der Spalten von A

Matrix × Matrix

$$A \cdot B = A \cdot (\underline{b}_1 \; \underline{b}_2 \dots \underline{b}_q)$$

$\underline{b}_1 \dots \underline{b}_q$ Spalten von B

$$= (A \cdot \underline{b}_1 \;\; A\underline{b}_2 \; \dots \; A\underline{b}_q)$$

Bsp Lineares Gleichungssystem

$$2x_1 - x_2 = 5$$
$$3x_1 + 2x_2 = 3$$

$$\Leftrightarrow \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \qquad A \cdot \underline{x} = \underline{b}$$

Achtung: 1.) $A \cdot B \neq B \cdot A$

2.) $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Satz (1) $A \cdot I = I \cdot A = A$ $\quad$ I .. Einheitsmatrix

(2) $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

(3) $(A+B) \cdot C = A \cdot C + B \cdot C$

(4) $A \cdot (B+C) = A \cdot B + A \cdot C$

(5) $\lambda \cdot (A \cdot B) = (\lambda A) \cdot B = A \cdot (\lambda \cdot B)$ $\quad$ λ .. Skalar

(6) $(A+B)^T = A^T + B^T$

(7) $(A \cdot B)^T = B^T \cdot A^T$

(8) $(\lambda \cdot A)^T = \lambda \cdot A^T$

Def $A \in K^{n \times n}$ invertierbar wenn

$$\Leftrightarrow \exists A^{-1} \in K^{n \times n} : A \cdot A^{-1} = A^{-1} \cdot A = I_n$$

Bsp $A = \begin{pmatrix} 2 & 3 \\ 0 & -1 \end{pmatrix} \in K^{2 \times 2}$ $\quad$ $A \cdot X = I_2$

$$\begin{pmatrix} 2 & 3 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} x_1 & x_3 \\ x_2 & x_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 2 & 3 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$2x_1 + 3x_2 = 1$ $\qquad$ $2x_3 + 3x_4 = 0$

$-x_2 = 0$ $\qquad$ $\cancel{0x_3} - x_4 = 1$

$x_1 = \frac{1}{2} \quad x_2 = 0$ $\qquad$ $x_3 = \frac{3}{2}, \; x_4 = -1$

$$X = A^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{3}{2} \\ 0 & -1 \end{pmatrix}$$

Bem $A \in K^{n \times n}$ invertierbar (= regulär) $\underline{b} \in K^n$

Lin. gl. Sys. $A \underline{x} = \underline{b} \Rightarrow \underbrace{A^{-1} \cdot (A \cdot \underline{x})}$ = $A^{-1} \cdot \underline{b}$

$(A^{-1} \cdot A) \cdot \underline{x}$

$E \cdot \underline{x} = \underline{x} = A^{-1} \cdot \underline{b}$

$\underline{x} = A^{-1} \cdot \underline{b}$

Satz $A \in K^{n \times n}$ invert $\Leftrightarrow$ Spalten bzw. Zeilen sind lin. unabh.

$\Leftrightarrow$ lin. Hülle der Spalten $= K^n$

$\Leftrightarrow$ Spalten bz Zeilen bilden Basis von K^n

$\underline{b} \in K^n$ mit Link. der Spalten von A

$\Leftrightarrow \exists \underline{x} \in K^n : A \cdot \underline{x} = \underline{b}$

$\Leftrightarrow A$ invertierbar

Def $A \in K^{m \times n}$

Rang von $A = rg(A)$ = Dimension der linearen Hülle der Spalten (Zeilen) von A

Bem $A \in K^{n \times n}$

A invertierbar $\Leftrightarrow rg(A) = n$

<u>Satz</u> $rg(A) = rg(A^T)$

[Spaltenrang = Zeilenrang]

<u>Elementare Umformung</u> e. Matrix $A \in K^{m \times n}$

(1) Multiplikation e. Spalte/Zeile mit Skalar $\neq 0$

(2) Vertauschen zweier Spalten / Zeilen

(3) Addieren eines Vielfachen e. Spalte/Zeile zu einer anderen.

<u>Satz</u> Elementare Umformungen verändern den Rang einer Matrix nicht

<u>Bsp</u>

$$A = \begin{pmatrix} 1 & 2 & -3 & 0 \\ 2 & 5 & 1 & 8 \\ -1 & -2 & 4 & 1 \\ 4 & 0 & 2 & 6 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 7 & 8 \\ -1 & 0 & 1 & 1 \\ 4 & -8 & 14 & 6 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 0 & 1 & 1 \\ 4 & -8 & 70 & 70 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 4 & -8 & 70 & 0 \end{pmatrix} \quad rg(A) = 3$$

$$\to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 20 & -8 & 70 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 90 & -8 & 70 & 0 \end{pmatrix} \to \to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Elementarmatrix - elementar umgeformte Einheitsmatrix

Bsp: $T_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$T_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \lambda \\ 0 & 0 & 1 \end{pmatrix}$ $\cdot\lambda$

$A \cdot T_1 = \begin{pmatrix} \underline{a}_1 & \underline{a}_2 & \underline{a}_3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} \underline{a}_1 & \underline{a}_3 & \underline{a}_2 \end{pmatrix}$

$A \cdot T_1 = \ldots = \begin{pmatrix} \underline{a}_1 & \underline{a}_2 & \lambda\underline{a}_2 + \underline{a}_3 \end{pmatrix}$ $\cdot\lambda$

Bsp:

$$A = \begin{array}{c} (-2) \\ \begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & 1 \\ 3 & 5 & 0 \end{pmatrix} \\ \hline \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \end{array} \rightarrow \begin{array}{c} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 3 & -1 & 0 \end{pmatrix} \\ \hline \begin{matrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \end{array} \rightarrow \begin{array}{c} (-2) \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 3 & 0 & -1 \end{pmatrix} \\ \hline \begin{matrix} 1 & 0 & -2 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{matrix} \end{array} \rightarrow \begin{array}{c} (+3) \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & -1 \end{pmatrix} \\ \hline \begin{matrix} 1 & 0 & -2 \\ 0 & 0 & 1 \\ 0 & 1 & -2 \end{matrix} \end{array}$$

$$\rightarrow \begin{array}{c} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \\ \hline \begin{matrix} -5 & 0 & -2 \\ +3 & 0 & 1 \\ -6 & 1 & -2 \end{matrix} \end{array} \rightarrow \begin{array}{c} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \hline \begin{matrix} -5 & 0 & 2 \\ 3 & 0 & -1 \\ -6 & 1 & 2 \end{matrix} \end{array} = A^{-1}$$

1. Zeile $A \cdot \underbrace{T_1 \cdot T_2 \cdot T_3 \cdot T_4 \cdot T_5}_{T} = I_3$ $\qquad A \cdot T = I_3$

$T = A^{-1}$

2. Zeile $I_3 \cdot \overbrace{T_1 \cdot T_2 \cdot T_3 \cdot T_4 \cdot T_5}^{T} = I_3 \cdot T = A^{-1}$

Bem $A \in K^{m \times n} \Rightarrow \exists\, T_1 \in K^{m \times m},\ T_2 \in K^{n \times n}$, invertierbar,

$$T_1 \cdot A \cdot T_2 = \left(\begin{array}{c|c} I_r & 0 \\ \hline 0 & 0 \end{array}\right) \qquad r = \text{rang}(A)$$

$(V,+,K)$ $(W,+,K)$ V Raum über K

Def $f: V \to W$ lineare Abbildung

$:\Longleftrightarrow$ (1) $f(\underline{x}+\underline{y}) = f(\underline{x}) + f(\underline{y})$ $\quad \underline{x}, \underline{y} \in V$

(2) $f(\lambda \cdot \underline{x}) = \lambda \cdot f(\underline{x})$ $\quad \lambda \in K$

Bsp $A \in K^{m \times n}$, $V = K^n$, $W = K^m$

$f(\underline{x}) := A \cdot \underline{x}$ ist lin. Abb.

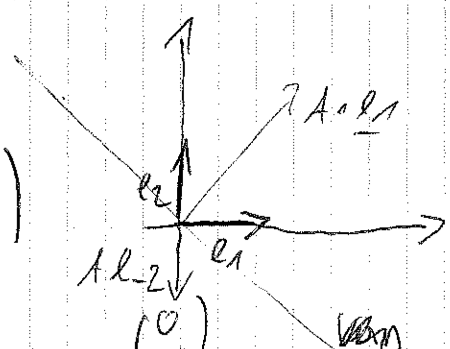
(1) $A \cdot (\underline{x}+\underline{y}) = A \cdot \underline{x} + A\, \underline{y}$ ✓

(2) $A \cdot (\lambda \underline{x}) = \lambda \cdot (A\, \underline{x})$ ✓

Bsp Drehung, Spiegelung, Drehstrecken sind lin. Abb (geometr.)

Bsp $A = \begin{pmatrix} 2 & 0 \\ 3 & -1 \end{pmatrix}$

$\underline{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad \underline{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



$A\underline{e}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\qquad A \cdot \underline{e}_2 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

1. Spalte v. A $\qquad$ 2. Spalte v. A

$$A\underline{x} = \begin{pmatrix} 2 & 0 \\ 3 & -1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1 \begin{pmatrix} 2 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

1. Beobachtung $A = (\underline{a}_1\ \underline{a}_2\ \dots\ \underline{a}_n) \Rightarrow A \cdot \underline{e}_j = \underline{a}_j$

Spalten

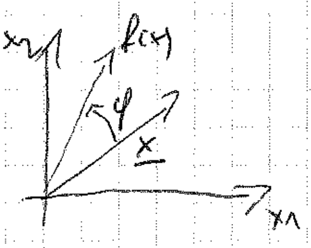
2. Beob. $\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \Rightarrow A\underline{x} = x_1 \cdot \underline{a}_1 + \dots + x_n \cdot a_n$

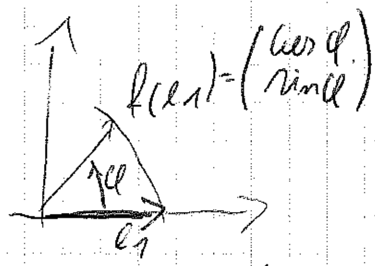
$= x_1 \cdot (A \underline{e}_1) + \dots + x_n (A \underline{e}_n)$

Satz $f: V \to W$ linear

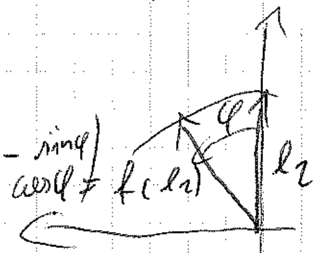
$\Rightarrow f(\lambda_1 \underline{v}_1 + \lambda_2 \underline{v}_2 + \dots + \lambda_n \cdot \underline{v}_n) = \lambda_1 \cdot f(v_1) + \dots \lambda_n \cdot f(v_n)$

Bsp Drehung um φ



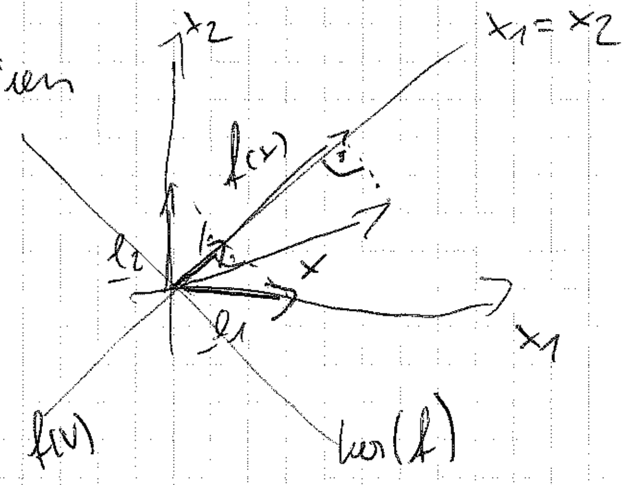


$x = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ $A = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$ $\begin{pmatrix} -\sin\varphi \\ \cos\varphi \end{pmatrix} = f(e_2)$



$A \cdot \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 5\cos\varphi - 2\sin\varphi \\ 5\sin\varphi + 2\cos\varphi \end{pmatrix}$

Bsp Projektion



$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

$\uparrow$ $f(e_1)$ $\uparrow$ $f(e_2)$

Def $f: V \to W$ linear

$f(V) = \{ f(\underline{x}) \mid \underline{x} \in V \} \leq W$

$\ker f = \{ \underline{x} \in V \mid f(\underline{x}) = \underline{0} \} \leq V$

$rg(f) = \dim f(V)$ .. Rang v. f

$def(f) = \dim(\ker(f))$ .. Defekt von f

Satz $f(\underline{x}) = A \cdot \underline{x} \Rightarrow rg(f) = rg(A)$

Satz $f: V \to W$ lin. $\Rightarrow rg(f) + def(f) = \dim(V)$

$\boxed{\text{RANGFORMEL}}$

Rem $\ker(f) = \{ \underline{x} \in V \mid f(x) = \underline{0} \} \leq V$

Bem $\ker f = ?$, $f(\underline{x}) = A \underline{x}$, $A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 2 & 2 \\ 3 & 1 & 5 \end{pmatrix}$

$\ker f = \{ x \in K^3 \mid f(x) = A \cdot \underline{x} = \underline{0} \} = \{ \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} \mid \lambda \in K \}$

$A \cdot \underline{x} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 2 & 2 \\ 3 & 1 & 5 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$x_1 + 2x_3 = 0$
$2x_1 + 2x_2 + 2x_3 = 0$
$3x_1 + x_2 + 5x_3 = 0$

$f(x) = \underline{0} \Rightarrow f(\lambda \underline{x}) = \lambda f(\underline{x}) = \lambda \cdot \underline{0} = \underline{0}$

$x_1 = 1$
$x_2 = -1$
$x_3 = -1$

Wh. 1. $A \in K^{m\times n}$ $\qquad f: K^n \to K^m$

$$x \mapsto f(x) = A\cdot\underline{x}$$

2. $f: K^n \to K^m$ lin.

$$A = \big(f(e_1)\ \ f(e_2)\ \cdots\ f(e_n)\big)$$

$$\Rightarrow f(\underline{x}) = A\cdot\underline{x}$$

$$\boxed{A \in K^{m\times n} \Leftrightarrow f: K^n \to K^m \text{ lin}}$$

$f: V \to W$, $g: W \to X$ linear

Hintereinander $\Rightarrow h = g\circ f: V \to X$ linear

$$f \leftrightarrow A \qquad g \leftrightarrow B \qquad g\circ f \leftrightarrow B\cdot A$$

$$(B\cdot A)\cdot\underline{x} = B\cdot(A\cdot\underline{x})$$

$$h(\underline{x}) = g(f(x))$$

Matrizenmultipl. kann interpr. werden als Hintereinanderausf. von lin. Abbildungen

Satz $f: V \to W$ lin.

$f(x_0) = b$

$\Rightarrow \{x \in V \mid f(\underline{x}) = \underline{b}\} = x_0 + \mathrm{Ker}(f)$ ... Nebenraum

Bew $\subseteq$ (1) $f(x) = b$, $f(x_0) = b$ $\Rightarrow f(\underline{x} - x_0) = f(\underline{x}) - f(\underline{x_0}) = \underline{b} - \underline{b} = \underline{0}$

$\Rightarrow \underline{x} - x_0 \in \mathrm{Ker}(f) \Rightarrow \underline{x} \in x_0 + \mathrm{Ker}(f)$

$\supseteq$ (2) $x \in x_0 + \mathrm{Ker}(f) \Rightarrow \underline{x} = x_0 + \underline{y}$

$\underline{y} \in \mathrm{Ker}(f)$ d.h. $f(\underline{y}) = \underline{0}$

$\Rightarrow f(\underline{x}) = f(\underline{x_0} + \underline{y}) = f(\underline{x_0}) + f(\underline{y})$

$= \underline{b} + \underline{0} = \underline{b}$

$\Rightarrow f(x) = b$

Lineares Gleichungssystem

Bsp

$$\begin{aligned} x_1 \quad\quad\;\; + x_3 &= 1 \\ 2x_1 + x_2 - 2x_3 &= 0 \\ 3x_1 + x_2 \quad\quad\;\; &= 1 \end{aligned} \iff \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$A \cdot \underline{x} = b$

$f(\underline{x}) = b$

Def $a_{ij} \in K$, $b_j \in K$ $\quad A = (a_{ij}) \in K^{m \times n}$

$\underline{b} = (b_j) \in K^m$

$\underline{x} = (x_i) = K^n$

$$m \left\{ \begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\ \vdots \qquad\qquad & \\ a_{m1}x_1 + \dots \qquad + a_{mn}x_n &= b_n \end{aligned} \right. \iff A\underline{x} = \underline{b}$$

n Unbekannte $\qquad$ LGS

erw. Koeff.-matrix $\Rightarrow (A \mid \underline{b})$

$$\begin{aligned} x_1 \quad\quad &+ x_3 = 1 \\ 2x_1 + x_2 &- 2x_3 = 0 \\ 3x_1 + x_2 \quad &= 1 \end{aligned}$$

(-2), (-3)

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 2 & 1 & -2 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right)$$

(-2), (-3)

element. Zeilenumf.

$x_1 \quad + x_3 = 1 \Rightarrow x_1 = 1$

$$\begin{aligned} x_2 - 4x_3 &= -2 \\ x_2 - 3x_3 &= -2 \end{aligned}$$

(-1)

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -4 & -2 \\ 0 & 1 & -3 & -2 \end{array}\right)$$

(-1)

$x_2 - 4x_3 = -2 \Rightarrow x_2 = -2$

$x_3 = 0 \Rightarrow x_3 = 0$

$\Leftrightarrow$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -4 & -2 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$\underline{x} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$

$x_1 = 1$
$x_2 = -2$
$x_3 = 0$

$\Leftrightarrow$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$$\begin{aligned} x_1 \quad\quad &+ x_3 = 1 \\ 2x_1 + x_2 &- 2x_3 = 0 \\ 3x_1 + x_2 &- x_3 = -1 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 2 & 1 & -2 & 0 \\ 3 & 1 & -1 & -1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -4 & -2 \\ 0 & 1 & -4 & -4 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -4 & -2 \\ 0 & 0 & 0 & -2 \end{array}\right) \Leftrightarrow 0 = -2$$

Unlösbar

$A \cdot \underline{x} = \underline{b}$ lösbar $\Leftrightarrow$
$\underline{b} \in$ lin. Hülle der Spalten von A

$rg(A\underline{b}) = rg(A)$

Bsp

$$\left(\begin{array}{cccc|c} 1 & +2 & -2 & 3 & 3 \\ 2 & 5 & 0 & 1 & 4 \\ 3 & 8 & 2 & -1 & 5 \\ 1 & 4 & 6 & -7 & -1 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & +2 & -2 & 3 & 3 \\ 0 & 1 & 4 & -5 & -2 \\ 0 & 2 & 8 & -10 & -4 \\ 0 & 2 & 8 & -10 & -4 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -2 & 3 & 3 \\ 0 & 1 & 4 & -5 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \quad (-2)$$

$\mathrm{rg} = 2$

$$\left(\begin{array}{cccc|c} \boxed{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}} & \boxed{\begin{matrix} -10 \\ 4 \end{matrix}} & \boxed{\begin{matrix} 13 \\ -5 \end{matrix}} & \begin{matrix} 7 \\ -2 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \end{array}\right) \Leftrightarrow \begin{aligned} x_1 \quad -10x_3 + 13x_4 &= 7 \\ x_2 + 4x_3 - 5x_4 &= -2 \end{aligned}$$

$x_3 = x_4 = 0 \quad x_1 = 7 \; x_2 = -2$

Parameter $x_3 = t_1$, $x_4 = t_2$

$$\begin{aligned} x_1 &= 7 + 10x_3 - 13x_4 = 7 + 10t_1 - 13t_2 \\ x_2 &= \qquad\qquad\qquad\quad = -2 - 4t_1 + 5t_2 \end{aligned}$$

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 7 \\ -2 \\ 0 \\ 0 \end{pmatrix} + t_1 \cdot \begin{pmatrix} 10 \\ -4 \\ 1 \\ 0 \end{pmatrix} + t_2 \cdot \begin{pmatrix} -13 \\ 5 \\ 0 \\ 1 \end{pmatrix}$$

$t_1, t_2 \in K$

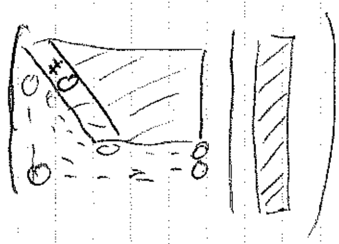
Spannen den Kern auf

Gaußsches Eliminationsverfahren

LGS $\quad A\underline{x} = \underline{b} \quad A \in K^{m \times n}, \; b \in K^m, \; x \in K^n$

$(A \mid \underline{b})$ $\xrightarrow{\text{elementare Zeilenumf.}^{*)}}$

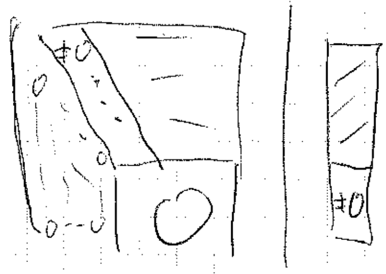
erw. System



*) + Spaltenvertauschungen in A

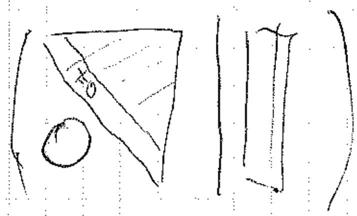
3 Fälle

1. Fall



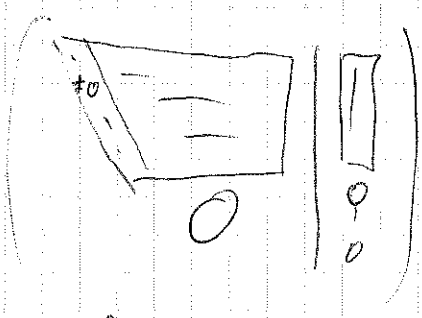
unlösbar

2. Fall



1 Lösung
eindeutig

3. Fall



mehrdeutige Lösung

$$\Rightarrow \left(E \quad c_1 \, .. \quad c_s \;\middle|\; d \right)$$

$$x = \begin{pmatrix} d \\ 0 \end{pmatrix} + t_1 \begin{pmatrix} -c_1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \dots + t_n \begin{pmatrix} -c_s \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$