

\oplus Negation $\begin{cases} I(p) = 0 \\ I(q) = 1 \\ I(r) = 1 \end{cases}$
 \oplus keine Negation $\begin{cases} I(p) = 1 \\ I(q) = 1 \\ I(r) = 1 \end{cases}$

erfüllbar wegen
 Belegungen!
 (wenn keine Klausel:
 \Rightarrow widerlegbar)

② Beweis!

$\neg (\forall x)(\exists y)(\forall z)(P(x, y) \vee Q(y, z)) \rightarrow ((\forall x)(\exists y) P(x, y) \vee (\exists y)(\exists z) Q(y, z))$ // Angabe
 $\neg ((\forall x)(\exists y)(\forall z)(P(x, y) \vee Q(y, z)) \rightarrow ((\forall x)(\exists y) P(x, y) \vee (\exists y)(\exists z) Q(y, z)))$ // Var. umbenennen
 $\neg ((\forall x)(\exists y)(\forall z)(P(x, y) \vee Q(y, z)) \rightarrow ((\forall x')(\exists y') P(x', y') \vee (\exists y'')(\exists z'') Q(y'', z'')))$ // \rightarrow weg!
 $\neg (\neg (\forall x)(\exists y)(\forall z)(P(x, y) \vee Q(y, z)) \vee ((\forall x')(\exists y') P(x', y') \vee (\exists y'')(\exists z'') Q(y'', z'')))$ // äußeres \neg weg!
 $\neg (\neg (\forall x)(\exists y)(\forall z)(P(x, y) \vee Q(y, z)) \wedge \neg ((\forall x')(\exists y') P(x', y') \vee (\exists y'')(\exists z'') Q(y'', z'')))$ // $\neg (A \vee B)$ weg!
 $(\forall x)(\exists y)(\forall z)(P(x, y) \vee Q(y, z)) \wedge \neg (\forall x')(\exists y') P(x', y') \wedge \neg (\exists y'')(\exists z'') Q(y'', z''))$ // \neg vor Quantoren weg!
 $(\forall x)(\exists y)(\forall z)(P(x, y) \vee Q(y, z)) \wedge (\exists x')(\forall y') \neg P(x', y') \wedge (\forall y'')(\forall z'') \neg Q(y'', z'')$ // \exists weg!
 $(\forall x)(\forall z)(P(x, f(x)) \vee Q(f(x), z)) \wedge (\forall y') \neg P(a, y') \wedge (\forall y'')(\forall z'') \neg Q(y'', z'')$ // \forall weg!
 $(P(x, f(x)) \vee Q(f(x), z)) \wedge \neg P(a, y') \wedge \neg Q(y'', z'')$ // Klauselmengen
 $C = \{ \{P(x, f(x)), Q(f(x), z)\}, \{ \neg P(a, y') \}, \{ \neg Q(y'', z'') \} \}$
 $\{x \mapsto a, y' \mapsto f(a)\}$ // Resolution
 $\{Q(f(a), z)\}$
 $\{y'' \mapsto f(a), z'' \mapsto z\}$
 $\{\}$ \Rightarrow widerlegbar
 m.g.u.: $\{x \mapsto a, y' \mapsto f(a), y'' \mapsto f(a), z'' \mapsto z\}$

① gültig!
 ② nicht erfüllbar!

3

$$\{P(a,b,c,x)\}^{\textcircled{1}}, \{\neg P(x,y,z,a)\}^{\textcircled{2}}, P(y,x,z,a)\}^{\textcircled{2}}, \{\neg P(x,y,z,a)\}^{\textcircled{2}}, P(z,y,x,a)\}^{\textcircled{2}}, \{\neg P(x,y,z,a)\}^{\textcircled{4}}, P(x,z,y,a)\}^{\textcircled{4}}, \\ \{\neg P(b,c,a,a)\}^{\textcircled{5}}$$

$$\begin{array}{l} \{P(a,b,c,x)\}^{\textcircled{1}} \quad \{\neg P(x,y,z,a)\}^{\textcircled{2}}, P(y,x,z,a)\}^{\textcircled{2}} \\ \hline \{P(b,a,c,a)\}^{\textcircled{6}} \quad \{\neg P(x,y,z,a)\}^{\textcircled{2}}, P(z,y,x,a)\}^{\textcircled{2}} \quad \{x \mapsto a, y \mapsto b, z \mapsto c\} \\ \hline \{P(c,a,b,a)\}^{\textcircled{7}} \quad \{\neg P(x,y,z,a)\}^{\textcircled{4}}, P(x,z,y,a)\}^{\textcircled{4}} \quad \{x \mapsto b, y \mapsto a, z \mapsto c\} \\ \hline \{P(c,b,a,a)\}^{\textcircled{5}} \quad \{x \mapsto c, y \mapsto a, z \mapsto b\} \\ \hline \{P(c,b,a,a)\}^{\textcircled{5}} \end{array}$$

Falscher Weg!

$$\begin{array}{l} \{P(a,b,c,x)\}^{\textcircled{1}} \quad \{\neg P(x,y,z,a)\}^{\textcircled{2}}, P(z,y,x,a)\}^{\textcircled{2}} \\ \hline \{P(c,b,a,a)\}^{\textcircled{6}} \quad \{\neg P(x,y,z,a)\}^{\textcircled{2}}, P(y,x,z,a)\}^{\textcircled{2}} \quad \{x \mapsto a, y \mapsto b, z \mapsto c\} \\ \hline \{P(b,c,a,a)\}^{\textcircled{7}} \quad \{\neg P(b,c,a,a)\}^{\textcircled{5}} \quad \{x \mapsto c, y \mapsto b, z \mapsto a\} \\ \hline \{P(b,c,a,a)\}^{\textcircled{7}} \quad \{\neg P(b,c,a,a)\}^{\textcircled{5}} \end{array}$$

$\{\}$

\Rightarrow Widerspruch!

m.g.u.: $\{x \mapsto a, y \mapsto b, z \mapsto c, x \mapsto c, (y \mapsto b) \rightarrow a\}$

③

$$\{P(a, c, b, x)\}, \{\neg P(x, y, z, a), P(y, x, z, a)\}, \{\neg P(x, y, z, a), P(z, y, x, a)\}, \\ \{\neg P(x, y, z, a), P(x, z, y, a)\}, \{\neg P(a, b, c, a)\}$$

$$\frac{\{P(a, c, b, x)\} \quad \{\neg P(x, y, z, a), P(x, z, y, a)\}}{\{P(a, b, c, a)\} \quad \{\neg P(a, b, c, a)\}} \quad \{x \mapsto a, y \mapsto c, z \mapsto b\}$$

$$\{P(a, b, c, a)\} \quad \{\neg P(a, b, c, a)\}$$

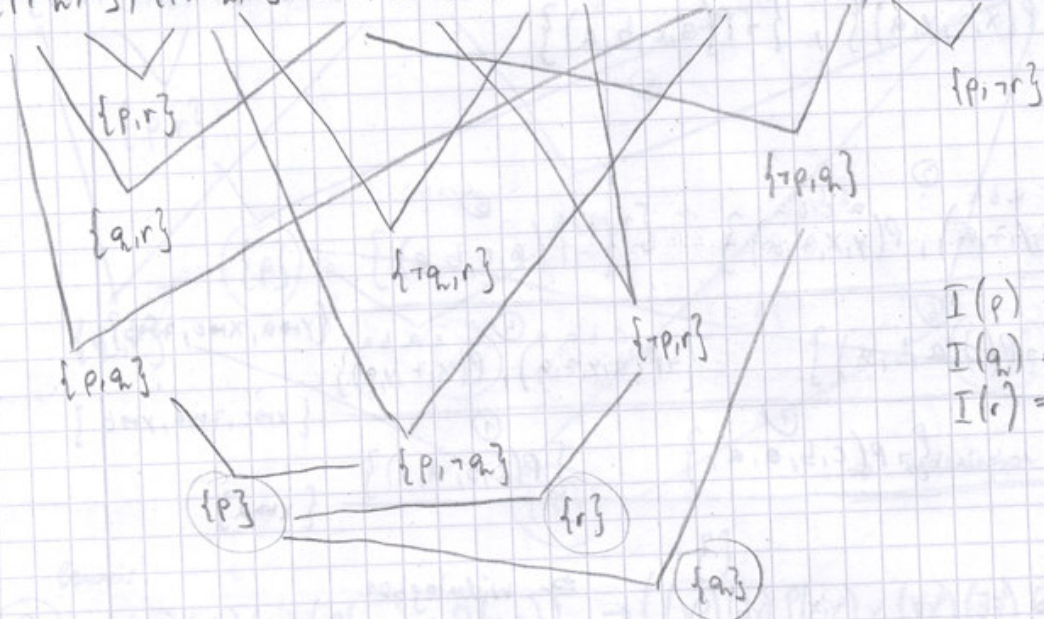
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⇒ widerlegbar!

m.g.U. $\{x \mapsto a, y \mapsto c, z \mapsto b\}$

①

$\{p, q, r\}, \{p, \neg q, r\}, \{\neg p, q, r\}, \{\neg p, \neg q, r\}, \{p, \neg r\}, \{q, \neg r\}, \{p, \neg q\}$



$$\begin{aligned} I(p) &= 1 \\ I(q) &= 1 \\ I(r) &= 1 \end{aligned}$$

②

Beweis:

$$\begin{aligned} & (\forall x)(\forall y)(\forall z) (P(x, y) \vee Q(y, z)) \rightarrow ((\forall x)(\exists y) P(x, y') \vee (\exists y'')(\exists z') Q(y'', z')) \\ & \neg \left(\neg \left((\forall x)(\forall y)(\forall z) (P(x, y) \vee Q(y, z)) \right) \rightarrow \left((\forall x)(\exists y) P(x, y') \vee (\exists y'')(\exists z') Q(y'', z') \right) \right) \\ & \neg \left(\neg \left((\forall x)(\forall y)(\forall z) (P(x, y) \vee Q(y, z)) \right) \wedge \neg \left((\forall x)(\exists y) P(x, y') \vee (\exists y'')(\exists z') Q(y'', z') \right) \right) \\ & (\forall x)(\forall y)(\forall z) (P(x, y) \vee Q(y, z)) \wedge \neg \left((\forall x)(\exists y) P(x, y') \vee (\exists y'')(\exists z') Q(y'', z') \right) \\ & \quad \wedge (\exists x)(\forall y) \neg P(x, y') \wedge (\forall y'')(\forall z') \neg Q(y'', z') \\ & \quad \wedge (\forall y) \neg P(a, y') \wedge (\forall y'')(\forall z') \neg Q(y'', z') \\ & (P(x, y) \vee Q(y, z)) \wedge \neg P(a, y') \wedge \neg Q(y'', z') \end{aligned}$$

$$C = \{ \{P(x, y), Q(y, z)\} ; \{ \neg P(a, y') \} ; \{ \neg Q(y'', z') \} \}$$

$$\begin{aligned} & \{x \mapsto a, y \mapsto y'\} \\ & \{a(y', z)\} \quad \{y' \mapsto y'', z \mapsto z'\} \\ & \{ \} \Rightarrow \text{widerlegbar!} \end{aligned}$$

$$\text{m.g.u. } \{x \mapsto a, y \mapsto y', y' \mapsto y'', z \mapsto z'\}$$

③

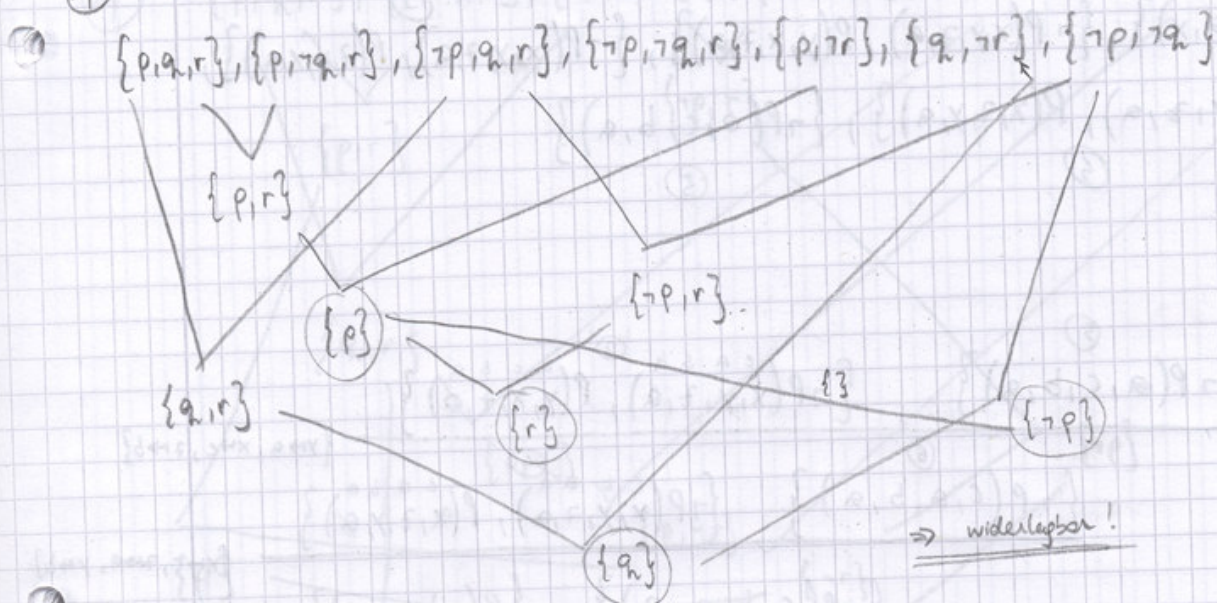
$$\begin{aligned} & \{P(c,b,a,x)\}^{(1)}, \{\neg P(x,y,z,a), P(y,x,z,a)\}^{(2)}, \{\neg P(x,y,z,a), P(z,y,x,a)\}^{(3)}, \\ & \{\neg P(x,y,z,a), P(x,z,y,a)\}^{(4)}, \{\neg P(a,c,b,a)\}^{(5)} \end{aligned}$$

$$\begin{aligned} & \frac{\{ \neg P(x,y,z,a), P(y,x,z,a) \}^{(2)} \quad \{ \neg P(a,c,b,a) \}^{(5)}}{\{ \neg P(c,a,b,a) \}^{(6)} \quad \{ \neg P(x,y,z,a), P(x,z,y,a) \}^{(4)}} \quad \{y \mapsto a, x \mapsto c, z \mapsto b\} \\ & \frac{\{ \neg P(c,a,b,a) \}^{(6)} \quad \{ \neg P(x,y,z,a), P(x,z,y,a) \}^{(4)}}{\{ \neg P(c,b,a,a) \}^{(7)} \quad \{ P(c,b,a,x) \}^{(1)}} \quad \{x \mapsto c, z \mapsto a, y \mapsto b\} \\ & \frac{\{ \neg P(c,b,a,a) \}^{(7)} \quad \{ P(c,b,a,x) \}^{(1)}}{\{ \} \quad \{x \mapsto a\}} \end{aligned}$$

{ } ⇒ widerlegbar

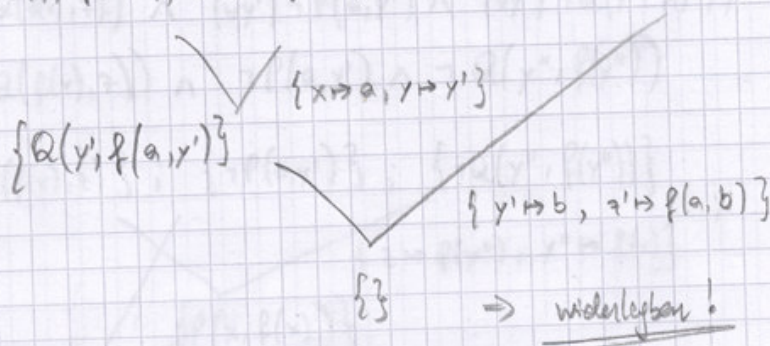
m.g.u. $\{y \mapsto a, x \mapsto c, z \mapsto b, x \mapsto c, z \mapsto a, y \mapsto b, x \mapsto a\}$

①



Beweis!

$$\begin{aligned}
 & (2) \quad (\forall x)(\forall y)(\exists z)(P(x,y) \vee Q(y,z)) \rightarrow ((\forall x)(\exists y)P(x,y) \vee (\forall y)(\exists z)Q(y,z)) \\
 & \neg \left(\neg (\forall x)(\forall y)(\exists z)(P(x,y) \vee Q(y,z)) \wedge \left(\neg ((\forall x)(\exists y)P(x,y) \vee (\forall y)(\exists z)Q(y,z)) \right) \right) \\
 & (\forall x)(\forall y)(\exists z)(P(x,y) \vee Q(y,z)) \wedge \neg ((\forall x)(\exists y)P(x,y) \vee (\forall y)(\exists z)Q(y,z)) \\
 & (\forall x)(\forall y)(\exists z)(P(x,y) \vee Q(y,z)) \wedge (\exists x')(\forall y')\neg P(x',y') \wedge (\exists y'')(\forall z')\neg Q(y'',z') \\
 & (\forall x)(\forall y) (P(x,y) \vee Q(y, f(x,y))) \wedge (\forall y')\neg P(a,y') \wedge (\forall z')\neg Q(b,z') \\
 & (P(x,y) \vee Q(y, f(x,y))) \wedge \neg P(a,y') \wedge \neg Q(b,z') \\
 & C = \{ \{P(x,y), Q(y, f(x,y))\} ; \{ \neg P(a,y') \} ; \{ \neg Q(b,z') \} \}
 \end{aligned}$$



$$m.g.u. \{x \mapsto a, y \mapsto y', y' \mapsto b, z' \mapsto f(a, b)\}$$

③

$$\{P(c, b, a, x)\}^{(1)}, \{\neg P(x, y, z, a), P(y, x, z, a)\}^{(2)}, \{\neg P(x, y, z, a), P(z, y, x, a)\}^{(3)}, \\ \{\neg P(x, y, z, a), P(x, z, y, a)\}^{(4)}, \{\neg P(a, c, b, a)\}^{(5)}$$

$$\{\neg P(a, c, b, a)\}^{(5)} \quad \{\neg P(x, y, z, a), P(y, x, z, a)\}^{(2)} \quad \{y \mapsto a, x \mapsto c, z \mapsto b\}$$

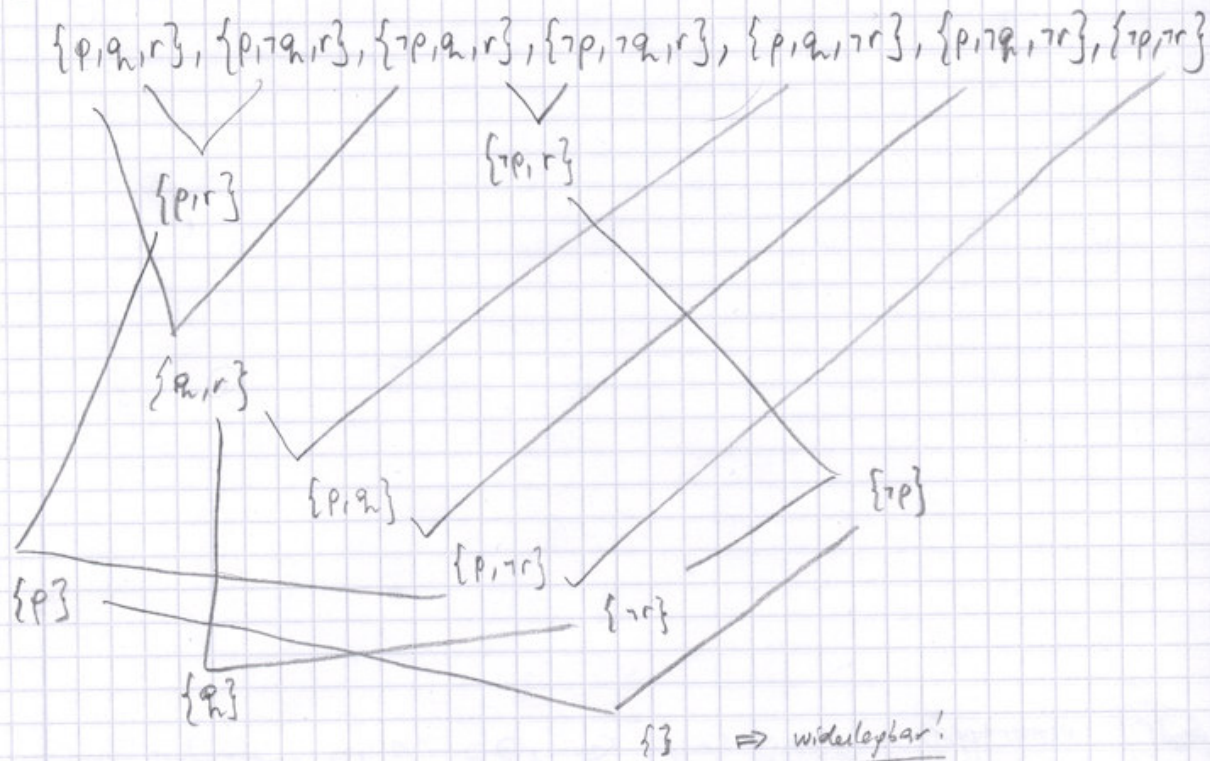
$$\{\neg P(c, a, b, a)\}^{(6)} \quad \{\neg P(x, y, z, a), P(x, z, y, a)\}^{(4)} \quad \{x \mapsto c, z \mapsto a, y \mapsto b\}$$

$$\{\neg P(c, b, a, a)\} \quad \{P(c, b, a, x)\} \quad \{x \mapsto a\}$$

$\{\} \Rightarrow$ widerlegen!

m. g. u. $\{y \mapsto a, x \mapsto c, z \mapsto b, x \mapsto c, z \mapsto a, y \mapsto b, x \mapsto a\}$

1



2

beweis!

$$\neg \left((\forall x)(\exists y)(\forall z) (P(x, y) \vee Q(y, z)) \rightarrow ((\forall x)(\exists y) P(x, y) \vee (\exists y)(\forall z) Q(y, z)) \right)$$

$$\neg \left(\neg (\forall x)(\exists y)(\forall z) (P(x, y) \vee Q(y, z)) \vee ((\forall x)(\exists y) P(x, y) \vee (\exists y)(\forall z) Q(y, z)) \right)$$

$$(\forall x)(\exists y)(\forall z) (P(x, y) \vee Q(y, z)) \wedge \neg ((\forall x)(\exists y) P(x, y) \vee (\exists y)(\forall z) Q(y, z))$$

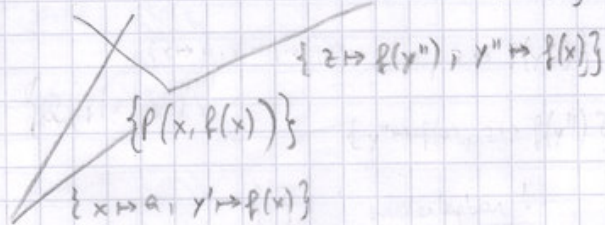
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$$\wedge (\exists x)(\forall y) \neg P(x, y) \wedge (\forall y)(\exists z) \neg Q(y, z)$$

$$(\forall x)(\forall z) (P(x, f(x)) \vee Q(f(x), z)) \wedge (\forall y') \neg P(a, y') \wedge (\forall y'') \neg Q(y'', f(y''))$$

$$(P(x, f(x)) \vee Q(f(x), z)) \wedge \neg P(a, y') \wedge \neg Q(y'', f(y''))$$

$$C = \{ \{ P(x, f(x)), Q(f(x), z) \} ; \{ \neg P(a, y') \} ; \{ \neg Q(y'', f(y'')) \} \}$$



{z}

⇒ widerlegbar!

$$m.g.u. \quad \{ z \mapsto f(y''), y'' \mapsto f(x), x \mapsto a, y' \mapsto f(x) \}$$

3

$$\{ \overset{1}{P(c,a,b,x)} \}, \{ \overset{2}{\neg P(x,y,z,a)}, P(y,x,z,a) \}, \{ \overset{3}{\neg P(x,y,z,a)}, P(z,y,x,a) \}, \{ \overset{4}{\neg P(x,y,z,a)}, P(x,z,y,a) \}, \\ \{ \overset{5}{\neg P(b,a,c,a)} \}$$

$$\frac{\{ \overset{5}{\neg P(b,a,c,a)} \} \quad \{ \overset{2}{\neg P(x,y,z,a)}, P(y,x,z,a) \}}{\neg P(a,b,c,a) \quad \{ \overset{3}{\neg P(x,y,z,a)}, P(z,y,x,a) \}} \quad \{ y \mapsto b, x \mapsto a, z \mapsto c \}$$

$$\frac{\neg P(a,b,c,a) \quad \{ \overset{3}{\neg P(x,y,z,a)}, P(z,y,x,a) \}}{\{ \overset{7}{\neg P(c,b,a,a)} \} \quad \{ \overset{4}{\neg P(x,y,z,a)}, P(x,z,y,a) \}} \quad \{ z \mapsto a, y \mapsto b, x \mapsto c \}$$

$$\frac{\{ \overset{7}{\neg P(c,b,a,a)} \} \quad \{ \overset{4}{\neg P(x,y,z,a)}, P(x,z,y,a) \}}{\{ \overset{2}{\neg P(c,a,b,a)} \} \quad \{ \overset{1}{P(c,a,b,x)} \}} \quad \{ x \mapsto c, z \mapsto b, y \mapsto a \}$$

$$\frac{\{ \overset{2}{\neg P(c,a,b,a)} \} \quad \{ \overset{1}{P(c,a,b,x)} \}}{\{ \}} \quad \{ x \mapsto a \}$$

\Rightarrow widersprüchlich!

m. g. u. $\{ y \mapsto b, x \mapsto a, z \mapsto c, z \mapsto a, y \mapsto b, \\ x \mapsto c, x \mapsto c, z \mapsto b, y \mapsto a, x \mapsto a \}$